8

DYNAMICS II: MOTION IN A PLANE

Conceptual Questions

8.1. For an object in uniform circular motion the speed is constant (that's what uniform means), the angular velocity is constant, and the magnitude of the net force is constant. The velocity and centripetal acceleration are both vectors whose magnitude is constant but whose direction changes.

8.2. The free-body diagram **(a)** is correct. The forces acting on the car at the bottom of the hill are the downward gravitational force and an upward normal force. The car can be considered to be in circular motion about a point above the bottom center of the valley, which requires a net force toward the center of the circle. In this case, the circle center is above the car, so the normal force is greater than the gravitational force.

8.3. $T_\text{c} > T_\text{a} = T_\text{d} > T_\text{b}$. Use $T = \dfrac{mv^2}{r}$. For **(a)**, $T_\text{a} = \dfrac{mv^2}{r}$. For **(b)**, $T_\text{b} = \dfrac{mv^2}{2r} = \dfrac{1}{2}T_\text{a}$. For **(c)**, $T_\text{c} = \dfrac{(2m)v^2}{r} = 2T_\text{a}$.

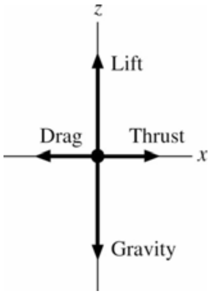
For **(d)**, $T_\text{d} = \dfrac{(2m)v^2}{2r} = T_\text{a}$.

8.4. The tension in the vine at the lowest part of Tarzan's swing is greater than the gravitational force on Tarzan. If Tarzan is at rest on the vine, just hanging, the tension in the vine is equal to the gravitational force on Tarzan. But when Tarzan is swinging he is in circular motion, with the center of the circle at the top end of the vine, and the vine must provide the additional centripetal force necessary to move him in a circle.

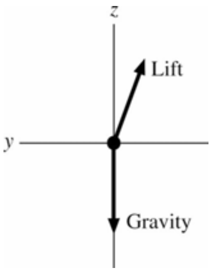
8.5. **(a)** The difference in the tension between A and B is due to the centripetal force $(F_r)_\text{net} = \dfrac{mv^2}{r}$. Since the velocity v is the same for both, the greater radius for B means that the tension in case A is greater than for case B.
(b) In this case, we use $(F_r)_\text{net} = m\omega^2 r$. The angular velocity ω is the same for both A and B, so the larger radius for B means that the tension is case A is less than the tension for case B.

8.6. Neither Sally nor Raymond is completely correct. Both have partially correct descriptions, but are missing key points. In order to speed up, there must be a nonzero acceleration parallel to the track. In order to move in a circle, there must be a nonzero centripetal acceleration. Since both of these are required, the net force points somewhere between the forward direction (parallel to the track) and the center of the circle.

8.7. **(a)** The plane is in dynamic equilibrium, so the net force on the plane is zero.
(b) The vertical forces cancel, and so do the horizontal forces, so the net force is zero. The plane is traveling in the positive x direction.



(c) As seen from behind, with the velocity and positive *x* direction into the page.

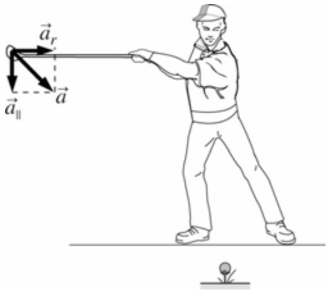


(d) The net force must be toward the center of the circle during a turn at constant speed and altitude. Note that the radial component of the lift force provides the centripetal force while the vertical component balances the gravitational force. The velocity is into the page.

8.8. Yes, the bug is weightless because it, like the projectile it is riding in, is in free fall around the planet.

8.9. When the gravitational force on the ball is greater than the required centripetal force $\frac{mv^2}{r}$, the ball is no longer in circular motion. As the figure shows, at the top of the circle the net force on the ball is $(F_r)_{\text{net}} = F_G + T = \frac{mv^2}{r}$. When the string goes slack, $T = 0$, leaving $F_G = \frac{mv^2}{r}$. If the velocity is not high enough to make this equality true, the equation above becomes an inequality, $F_G > \frac{mv^2}{r}$, and the ball begins to fall downward since the net force downward is greater than the centripetal force required for circular motion.

8.10.



The golfer is swinging the club in circular motion. The club is speeding up as it swings. This motion requires a linear acceleration in the direction of motion of the club to speed it up and a centripetal acceleration to maintain circular motion. The vector sum yields a total acceleration pointing approximately toward the golfer's feet **(c)**, as shown in the figure.

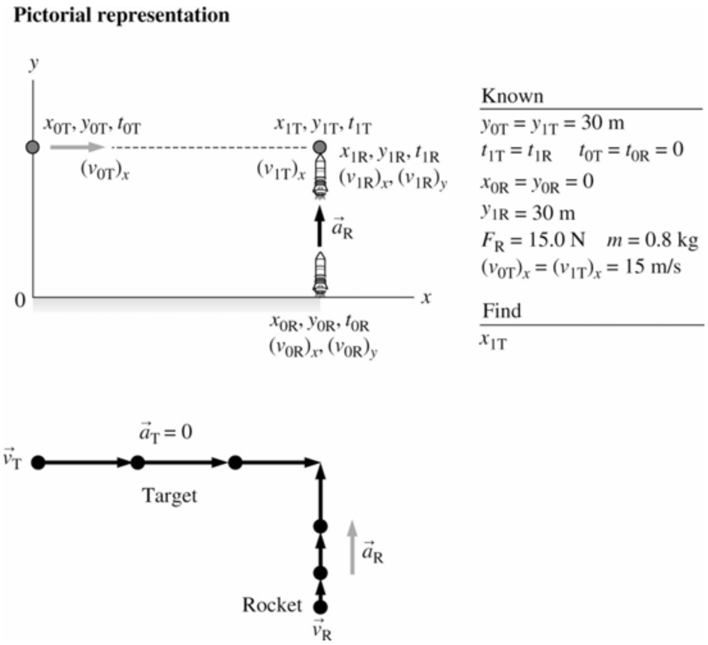
Exercises and Problems

Exercises

Section 8.1 Dynamics in Two Dimensions

8.1. Model: The model rocket and the target will be treated as particles. The kinematics equations in two dimensions apply.

Visualize:



Solve: For the rocket, Newton's second law along the *y*-direction is

$$(F_{\text{net}})_y = F_R - mg = ma_R$$

$$\Rightarrow a_R = \frac{1}{m}(F_R - mg) = \frac{1}{0.8\text{ kg}}[15\text{ N} - (0.8\text{ kg})(9.8\text{ m/s}^2)] = 8.95\text{ m/s}^2$$

Using the kinematic equation $y_{1R} = y_{0R} + (v_{0R})_y(t_{1R} - t_{0R}) + \frac{1}{2}a_R(t_{1R} - t_{0R})^2$,

$$30\text{ m} = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(8.95\text{ m/s}^2)(t_{1R} - 0\text{ s})^2 \Rightarrow t_{1R} = 2.589\text{ s}$$

For the target (noting $t_{1T} = t_{1R}$),

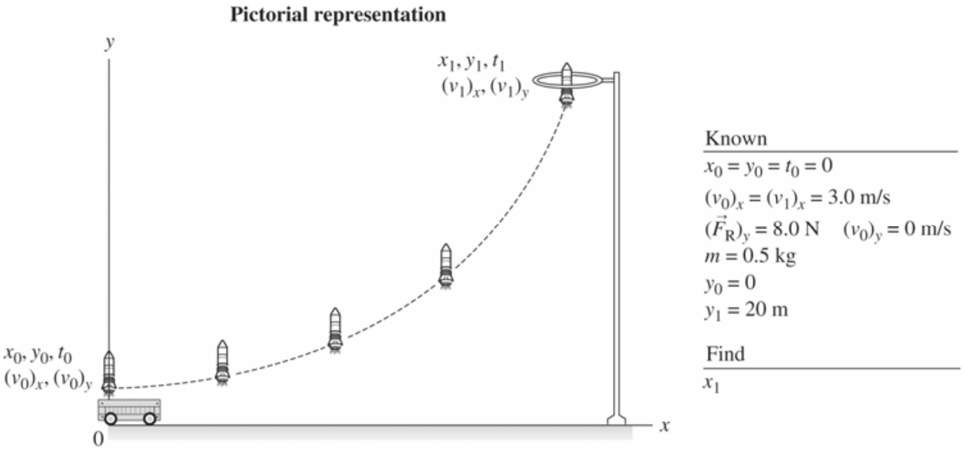
$$x_{1T} = x_{0T} + (v_{0T})_x(t_{1T} - t_{0T}) + \tfrac{1}{2}a_T(t_{1T} - t_{0T})^2 = 0\text{ m} + (15\text{ m/s})(2.589\text{ s} - 0\text{ s}) + 0\text{ m} = 39\text{ m}$$

You should launch when the target is 39 m away.

Assess: The rocket is to be fired when the target is at x_{0T}. For a net acceleration of approximately 9 m/s^2 in the vertical direction and a time of 2.6 s to cover a vertical distance of 30 m, a horizontal distance of 39 m is reasonable.

8.2. Model: The model rocket will be treated as a particle. Kinematic equations in two dimensions apply. Air resistance is neglected.

Visualize:



The horizontal velocity of the rocket is equal to the speed of the cart, which is 3.0 m/s.

Solve: For the rocket, Newton's second law along the *y*-direction is:

$$(F_{\text{net}})_y = F_R - mg = ma_R \Rightarrow a_y = \frac{1}{0.5\text{ kg}}[(8.0\text{ N}) - (0.5\text{ kg})(9.8\text{ m/s}^2)] = 6.2\text{ m/s}^2$$

Thus using $y_1 = y_0 + (v_0)_y(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$(20\text{ m}) = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(6.2\text{ m/s}^2)(t_{1R} - 0\text{ s})^2 \Rightarrow (20\text{ m}) = (3.1\text{ m/s}^2)t_1^2 \Rightarrow t_1 = 2.54\text{ s}$$

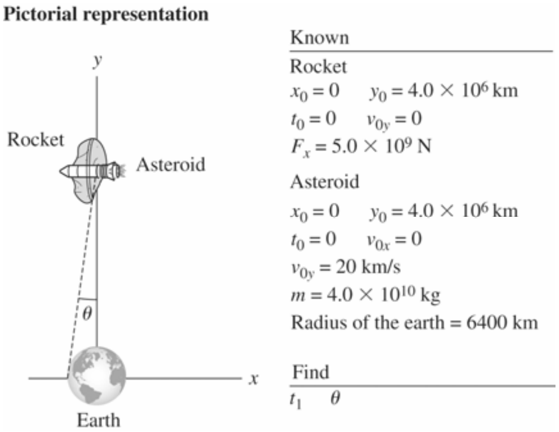
Since t_1 is also the time for the rocket to move horizontally up to the hoop,

$$x_1 = x_0 + (v_0)_x(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 = 0\text{ m} + (3.0\text{ m/s})(2.54\text{ s} - 0\text{ s}) + 0\text{ m} = 7.6\text{ m}$$

Assess: In view of the rocket's horizontal speed of 3.0 m/s and its vertical thrust of 8.0 N, the above-obtained value for the horizontal distance is reasonable.

8.3. Model: The asteroid and the giant rocket will be treated as particles undergoing motion according to the constant-acceleration equations of kinematics.

Visualize:


Pictorial representation
y
Rocket
Asteroid
θ
x
Earth
Known
Rocket
$x_0 = 0$ $y_0 = 4.0 \times 10^6$ km
$t_0 = 0$ $v_{0y} = 0$
$F_x = 5.0 \times 10^9$ N
Asteroid
$x_0 = 0$ $y_0 = 4.0 \times 10^6$ km
$t_0 = 0$ $v_{0x} = 0$
$v_{0y} = 20$ km/s
$m = 4.0 \times 10^{10}$ kg
Radius of the earth = 6400 km
Find
t_1 θ

Solve: **(a)** The time it will take the asteroid to reach the earth is

$$\frac{\text{displacement}}{\text{velocity}} = \frac{4.0\times10^6\text{ km}}{20\text{ km/s}} = 2.0\times10^5\text{ s} = 56\text{ h}$$

(b) The angle of a line that just misses the earth is

$$\tan\theta = \frac{R}{y_0} \Rightarrow \theta = \tan^{-1}\left(\frac{R}{y_0}\right) = \tan^{-1}\left(\frac{6400\text{ km}}{4.0\times10^6\text{ km}}\right) = 0.092°$$

(c) When the rocket is fired, the horizontal acceleration of the asteroid is

$$a_x = \frac{5.0\times10^9\text{ N}}{4.0\times10^{10}\text{ kg}} = 0.125\text{ m/s}^2$$

(Note that the mass of the rocket is much smaller than the mass of the asteroid and can therefore be ignored completely.) The velocity of the asteroid after the rocket has been fired for 300 s is

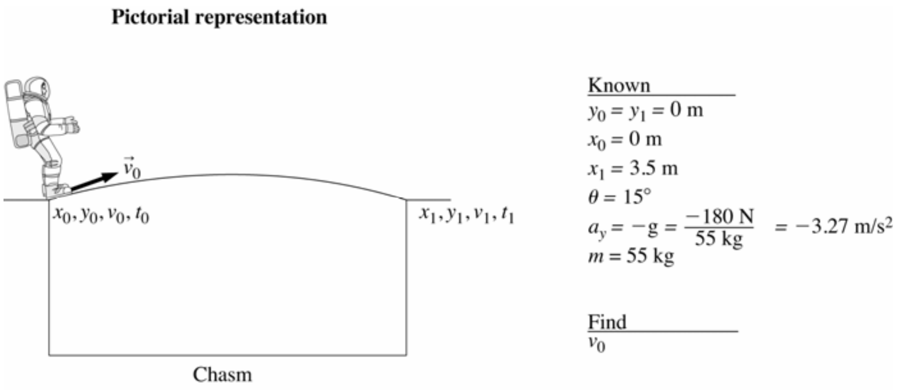
$$v_x = v_{0x} + a_x(t - t_0) = 0\text{ m/s} + (0.125\text{ m/s}^2)(300\text{ s} - 0\text{ s}) = 37.5\text{ m/s}$$

After 300 s, the vertical velocity is $v_y = 2\times10^4$ m/s and the horizontal velocity is $v_x = 37.5$ m/s. The deflection due to this horizontal velocity is

$$\tan\theta = \frac{v_x}{v_y} \Rightarrow \theta = \tan^{-1}\left(\frac{37.5\text{ m/s}}{2\times10^4\text{ m/s}}\right) = 0.107°$$

That is, the earth is saved.
Assess: Because the initial distance is so large the angle is quite small.

8.4. Model: Model the astronaut as a particle.
Visualize: The astronaut has constant vertical acceleration (–*g*) and constant horizontal velocity.



Solve: Analyze each direction separately. Start with the *y* direction to find the time she reaches the other side of the chasm.

$$y_1 = y_0 + (v_0\sin\theta)\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$0\text{ m} = 0\text{ m} + (v_0\sin\theta)t_1 - \tfrac{1}{2}gt_1^2 \Rightarrow t_1 = \frac{2v_0\sin\theta}{g}$$

Now use that *t* to determine the necessary velocity in the *x*-direction.

$$x_1 = x_0 + (v_0\cos\theta)\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$x_1 = 0\text{ m} + (v_0\cos\theta)t_1 + 0\text{ m} = (v_0\cos\theta)\frac{2v_0\sin\theta}{g}$$

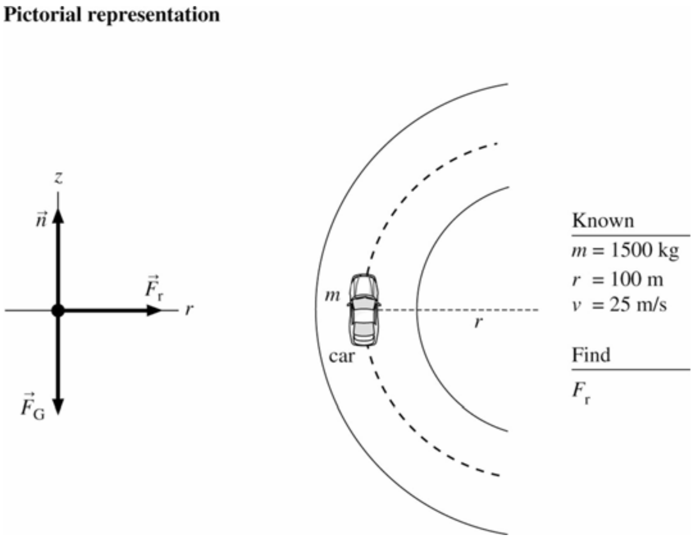
$$\Rightarrow v_0 = \sqrt{\frac{x_1 g}{2\sin\theta\cos\theta}} = \sqrt{\frac{(3.5\text{ m})(3.27\text{ m/s}^2)}{2\sin15°\cos15°}} = 4.8\text{ m/s}$$

Assess: This speed is achievable by an astronaut even in a bulky suit.

Section 8.2 Uniform Circular Motion

8.5. Model: We are using the particle model for the car in uniform circular motion on a flat circular track. There must be friction between the tires and the road for the car to move in a circle.

Visualize:



Solve: The centripetal acceleration is

$$a_r = \frac{v^2}{r} = \frac{(25\ \text{m/s})^2}{100\ \text{m}} = 6.25\ \text{m/s}^2$$

The acceleration points to the center of the circle, so the net force is

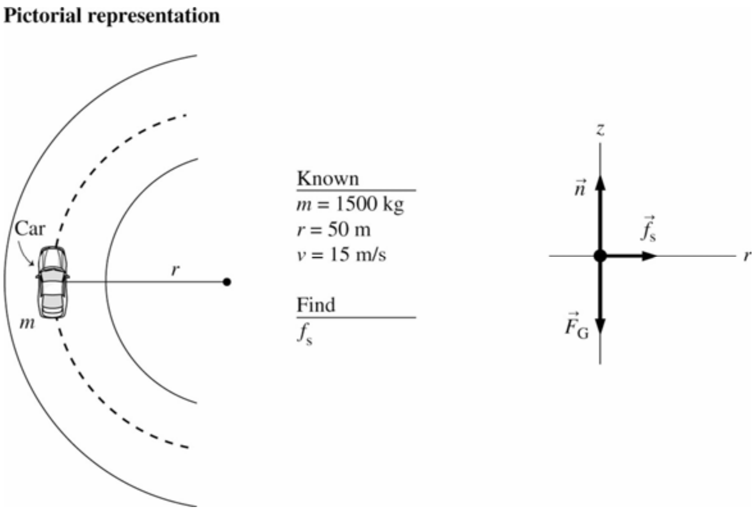
$$\vec{F}_r = m\vec{a} = (1500\ \text{kg})(6.25\ \text{m/s}^2,\ \text{toward center})$$
$$= (9380\ \text{N, toward center}) \approx (9400\ \text{N, toward center})$$

This force is provided by static friction

$$f_\text{s} = F_r = 9.4\ \text{kN}$$

8.6. Model: We will use the particle model for the car which is in uniform circular motion.

Visualize:



Solve: The centripetal acceleration of the car is

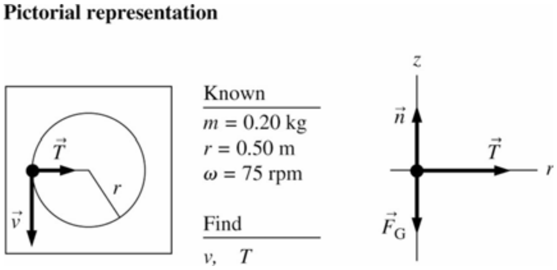
$$a_r = \frac{v^2}{r} = \frac{(15 \text{ m/s})^2}{50 \text{ m}} = 4.5 \text{ m/s}^2$$

The acceleration is due to the force of static friction. The force of friction is $f_s = ma_r = (1500 \text{ kg})(4.5 \text{ m/s}^2) = 6750 \text{ N} = 6.8 \text{ kN}$.

Assess: The model of static friction is $(f_s)_{\text{max}} = n\mu_s = mg\mu_s \approx mg \approx 15{,}000 \text{ N}$ since $\mu_s \approx 1$ for a dry road surface. We see that $f_s < (f_s)_{\text{max}}$, which is reasonable.

8.7. Model: Treat the block as a particle attached to a massless string that is swinging in a circle on a frictionless table.

Visualize:



Solve: (a) The angular velocity and speed are

$$\omega = 75\frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 471.2 \text{ rad/min} \quad v_t = r\omega = (0.50 \text{ m})(471.2 \text{ rad/min}) \times \frac{1 \text{ min}}{60 \text{ s}} = 3.93 \text{ m/s}$$

The tangential velocity is 3.9 m/s.

(b) The radial component of Newton's second law is

$$\sum F_r = T = \frac{mv^2}{r}$$

Thus

$$T = (0.20 \text{ kg})\frac{(3.93 \text{ m/s})^2}{0.50 \text{ m}} = 6.2 \text{ N}$$

8.8. Solve: Newton's second law is $F_r = ma_r = mr\omega^2$. Substituting into this equation yields:

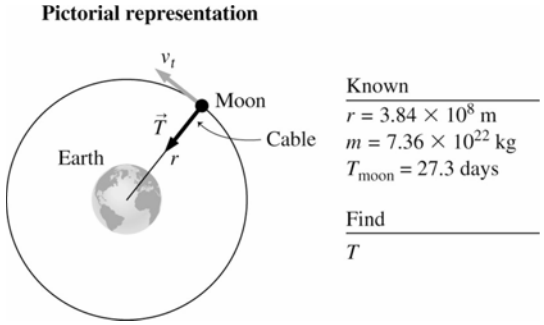
$$\omega = \sqrt{\frac{F_r}{mr}} = \sqrt{\frac{8.2 \times 10^{-8} \text{ N}}{(9.1 \times 10^{-31} \text{ kg})(5.3 \times 10^{-11} \text{ m})}}$$

$$= 4.37 \times 10^{16} \text{ rad/s } = 4.37 \times 10^{16} \frac{\text{rad}}{\text{s}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 6.6 \times 10^{15} \text{ rev/s}$$

Assess: This is a very high number of revolutions per second.

8.9. Model: The motion of the moon around the earth will be treated through the particle model. The circular motion is uniform.

Visualize:



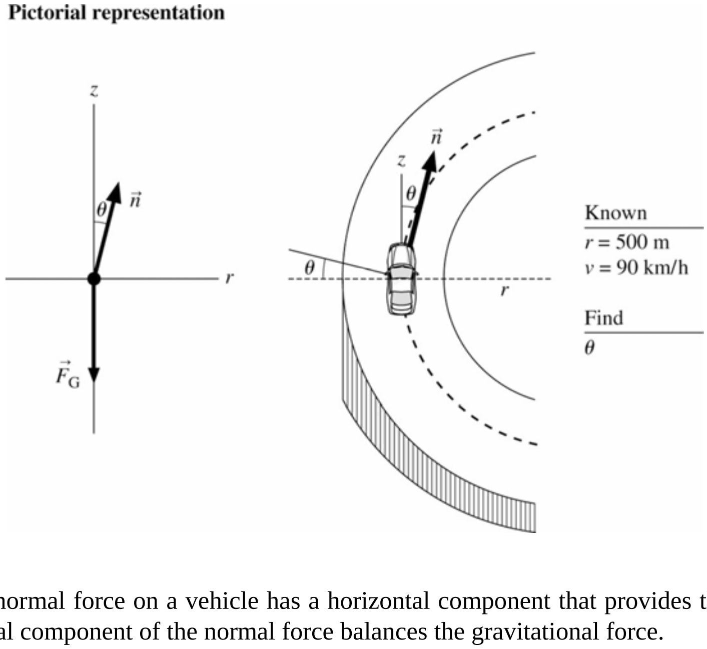
Solve: The tension in the cable provides the centripetal acceleration. Newton's second law is

$$\sum F_r = T = mr\omega^2 = mr\left(\frac{2\pi}{T_{\text{moon}}}\right)^2$$

$$= (7.36\times10^{22}\text{ kg})(3.84\times10^{8}\text{ m})\left[\frac{2\pi}{27.3\text{ days}}\times\frac{1\text{ day}}{24\text{ h}}\times\frac{1\text{ h}}{3600\text{ s}}\right]^2 = 2.01\times10^{20}\text{ N}$$

Assess: This is a tremendous tension, but clearly understandable in view of the moon's large mass and the large radius of circular motion around the earth. This is the same answer we'll get later with Newton's law of universal gravitation.

8.10. Model: The vehicle is to be treated as a particle in uniform circular motion.
Visualize:



On a banked road, the normal force on a vehicle has a horizontal component that provides the necessary centripetal acceleration. The vertical component of the normal force balances the gravitational force.
Solve: From the physical representation of the forces in the *r-z* plane, Newton's second law can be written

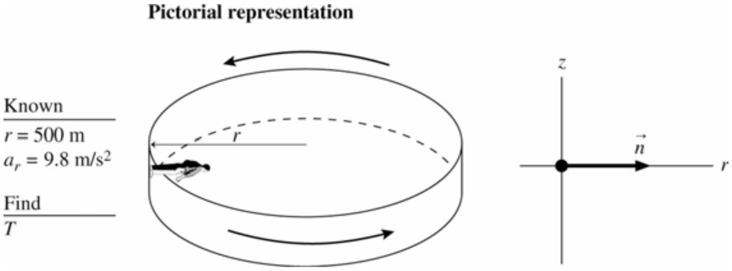
$$\sum F_r = n\sin\theta = \frac{mv^2}{r} \quad \sum F_z = n\cos\theta - mg = 0 \Rightarrow n\cos\theta = mg$$

Dividing the two equations and making the conversion $90 \text{ km/h} = 25 \text{ m/s}$ yields:

$$\tan\theta = \frac{v^2}{rg} = \frac{(25 \text{ m/s})^2}{(9.8 \text{ m/s}^2)500 \text{ m}} = 0.128 \Rightarrow \theta = 7.3°$$

Assess: Such a banking angle for a speed of approximately 55 mph is clearly reasonable and within our experience as well.

8.11. Model: Model the person as a particle in uniform circular motion.
Visualize:



Solve: The only force acting on the passengers is the normal force of the wall. Newton's second law along the r-axis is:

$$\sum F_r = n = mr\omega^2$$

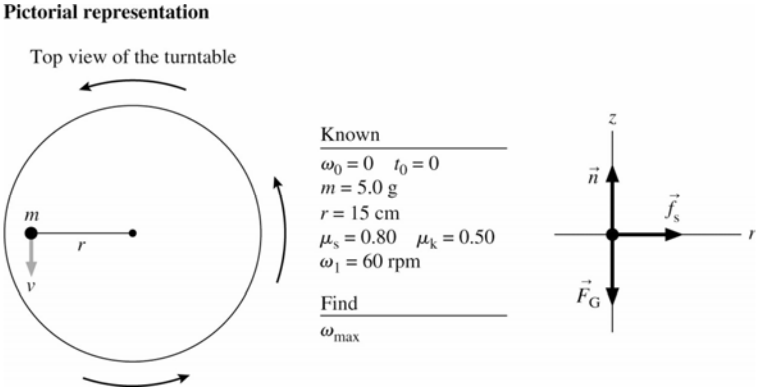
To create "normal" gravity, the normal force by the inside surface of the space station equals mg. Therefore,

$$mg = mr\omega^2 \Rightarrow \omega = \frac{2\pi}{T} = \sqrt{\frac{g}{r}} \Rightarrow T = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{500 \text{ m}}{9.8 \text{ m/s}^2}} = 45 \text{ s}$$

Assess: This is a fast rotation. The tangential speed is

$$v = \frac{2\pi r}{T} = \frac{2\pi(500 \text{ m})}{45 \text{ s}} = 70 \text{ m/s} \approx 140 \text{ mph}$$

8.12. Model: Use the particle model and static friction model for the coin, which is undergoing circular motion.
Visualize:



Solve: The force of static friction is $f_s = \mu_s n = \mu_s mg$. This force is equivalent to the maximum centripetal force that can be applied without sliding. That is,

$$\mu_s mg = m\frac{v_t^2}{r} = m(r\omega_{\max}^2) \Rightarrow \omega_{\max} = \sqrt{\frac{\mu_s g}{r}} = \sqrt{\frac{(0.80)(9.8 \text{ m/s}^2)}{0.15 \text{ m}}} = 7.23 \text{ rad/s}$$

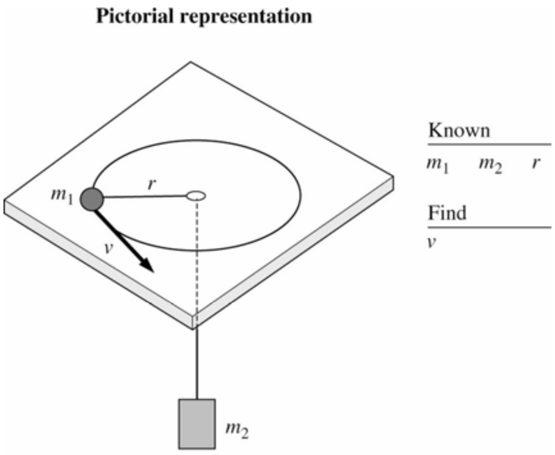
$$= 7.23\frac{\text{rad}}{\text{s}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}} = 69 \text{ rpm}$$

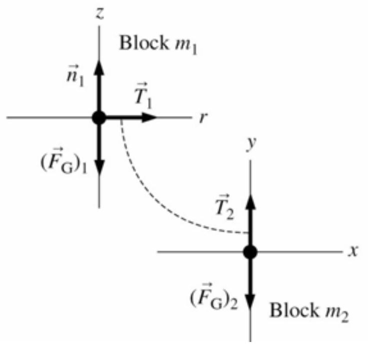
So, the coin will stay still on the turntable.

Assess: A rotational speed of approximately 1 rev per second for the coin to stay stationary seems reasonable.

8.13. Model: Masses m_1 and m_2 are considered particles. The string is assumed to be massless.

Visualize:





Solve: The tension in the string causes the centripetal acceleration of the circular motion. If the hole is smooth, it acts like a pulley. Thus tension forces $\vec{T}_1$ and $\vec{T}_2$ act as if they were an action/reaction pair. Mass m_1 is in circular motion of radius *r*, so Newton's second law for m_1 is

$$\sum F_r = T_1 = \frac{m_1 v^2}{r}$$

Mass m_2 is at rest, so the *y*-equation of Newton's second law is

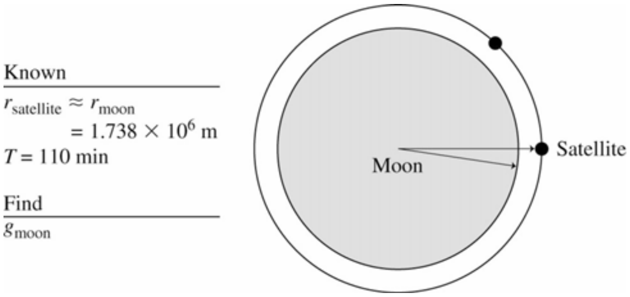
$$\sum F_y = T_2 - m_2 g = 0\text{ N} \Rightarrow T_2 = m_2 g$$

Newton's third law tells us that $T_1 = T_2$. Equating the two expressions for these quantities:

Section 8.3 Circular Orbits

8.14. Model: The satellite is considered to be a particle in uniform circular motion around the moon.

Visualize:



Solve: The radius of the moon is 1.738×10^6 m and the satellite's distance from the center of the moon is the same quantity. The angular velocity of the satellite is

$$\omega=\frac{2\pi}{T}=\frac{2\pi\text{ rad}}{110\text{ min}}\times\frac{1\text{ min}}{60\text{ s}}=9.52\times10^{-4}\text{ rad/s}$$

and the centripetal acceleration is

$$a_r=r\omega^2=(1.738\times10^6\text{ m})(9.52\times10^{-4}\text{ rad/s})^2=1.58\text{ m/s}^2$$

The acceleration of a body in orbit is the local "*g*" experienced by that body.

8.15. Model: The earth is considered to be a particle in uniform circular motion around the sun.

Solve: The earth orbits the sun in 365 days and is 1.5×10^{11} m from the sun. The angular velocity and centripetal acceleration are

$$\omega=\frac{2\pi\text{ rad}}{365\text{ days}}\times\frac{1\text{ day}}{24\text{ h}}\times\frac{1\text{ h}}{3600\text{ s}}=2.0\times10^{-7}\text{ rad/s}$$

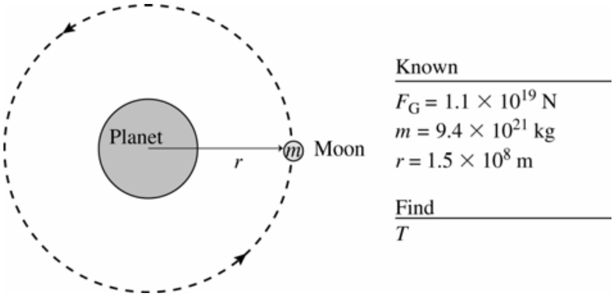
$$a_r=g=r\omega^2=(1.5\times10^{11}\text{ m})(2.0\times10^{-7}\text{ rad/s})^2=6.0\times10^{-3}\text{ m/s}^2$$

Assess: The smallness of this acceleration due to gravity is essentially due to the large earth-sun distance.

$$\frac{m_1v^2}{r}=m_2g\Rightarrow v=\sqrt{\frac{m_2rg}{m_1}}$$

8.16. Model: Model the moon as a particle in uniform circular motion.

Visualize: The moon has an acceleration of $a=v^2/r$. By the second law $a=F/m$, so set the two equal to each other.

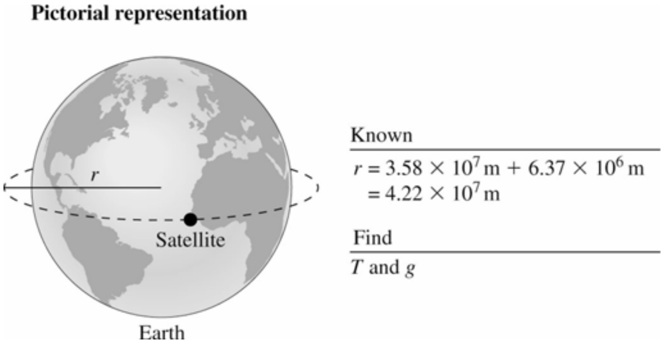


Solve: $F/m = v^2/r \Rightarrow v = \sqrt{\frac{rF}{m}}$. The orbital period is the circumference of the orbit divided by the speed.

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{rF}{m}}} = 2\pi\sqrt{\frac{rm}{F}} = 2\pi\sqrt{\frac{(1.5\times10^{8}\ \text{m})(9.4\times10^{21}\ \text{kg})}{1.1\times10^{19}\ \text{N}}} = 2.25\times10^{6}\ \text{s} = 26\ \text{d}$$

Assess: 26 earth days is a reasonable period for a moon; it is just a couple of days less than the orbital period for our own moon.

8.17. Model: Assume the particle model for the satellite in circular motion.
Visualize:



To be in a geosynchronous orbit means rotating at the same rate as the earth, which is 24 hours for one complete rotation. Because the altitude of the satellite is $3.58\times10^{7}\ \text{m}$, $r = 3.58\times10^{7}\ \text{m}$, $r_e = 3.58\times10^{7}\ \text{m} + 6.37\times10^{6}\ \text{m} = 4.22\times10^{7}\ \text{m}$.

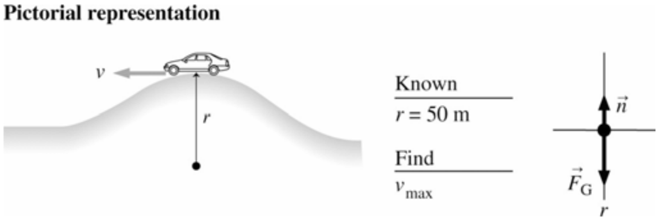
Solve: **(a)** The period (T) of the satellite is 24.0 hours.
(b) The acceleration due to gravity is

$$g = a_r = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2 = (4.22\times10^{7}\,\text{m})\left(\frac{2\pi}{24.0\ \text{hr}}\times\frac{1\ \text{hr}}{3600\ \text{s}}\right)^2 = 0.223\ \text{m/s}^2$$

(c) There is no normal force on a satellite, so the weight is zero. It is in free fall.

Section 8.4 Reasoning About Circular Motion

8.18. Model: Use the particle model for the car which is undergoing circular motion.
Visualize:



Solve: The car is in circular motion with the center of the circle below the car. Newton's second law at the top of the hill is

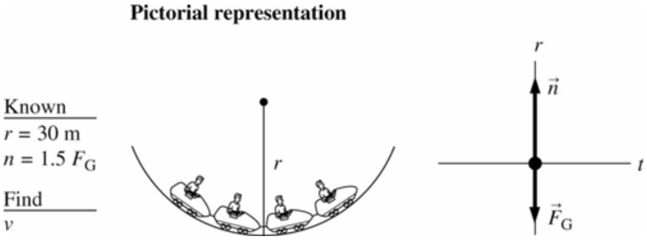
$$\sum F_r = (F_G)_r - n_r = mg - n = ma_r = \frac{mv^2}{r} \Rightarrow v^2 = r\left(g - \frac{n}{m}\right)$$

Maximum speed is reached when $n = 0$ and the car is beginning to lose contact with the road.

$$v_{\text{max}} = \sqrt{rg} = \sqrt{(50\text{ m})(9.8\text{ m/s}^2)} = 22\text{ m/s}$$

Assess: A speed of 22 m/s is equivalent to 49 mph, which seems like a reasonable value.

8.19. Model: The passengers are particles in circular motion.
Visualize:

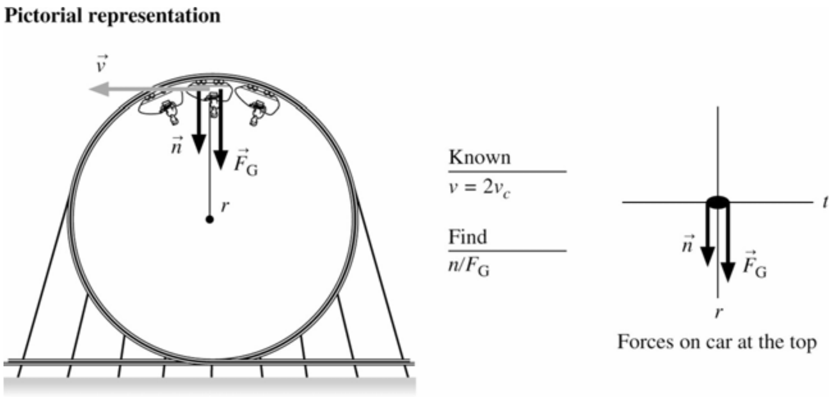


Solve: The center of the circle of motion of the passengers is directly above them. There must be a net force pointing up that provides the needed centripetal acceleration. The normal force on the passengers is their weight. Ordinarily their weight is F_G, so if their weight increases by 50%, $n = 1.5\,F_G$. Newton's second law at the bottom of the dip is

$$\sum F_r = n - F_G = (1.5 - 1)F_G = 0.5mg = \frac{mv^2}{r}$$
$$\Rightarrow v = \sqrt{0.5gr} = \sqrt{0.5(9.8\text{ m/s}^2)(30\text{ m})} = 12\text{ m/s}$$

Assess: A speed of 12.1 m/s is 27 mph, which seems very reasonable.

8.20. Model: Model the roller coaster car as a particle at the top of a circular loop-the-loop undergoing uniform circular motion.
Visualize:



Notice that the r-axis points downward, toward the center of the circle.

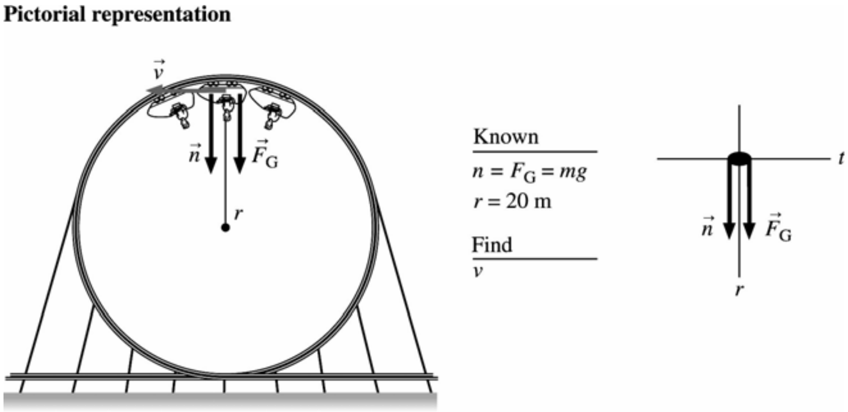
Solve: The critical speed occurs when $\vec{n}$ goes to zero and $\vec{F}_G$ provides all the centripetal force pulling the car in the vertical circle. At the critical speed $mg = mv_c^2/r$, therefore $v_c = \sqrt{rg}$. Since the car's speed is twice the critical speed, $v_t = 2v_c$ and the centripetal force is

$$\sum F_r = n + F_G = \frac{mv^2}{r} = \frac{m(4v_c^2)}{r} = \frac{m(4rg)}{r} = 4mg$$

Thus the normal force is $n = 3\,mg$. Consequently, $n/F_G = 3$.

8.21. Model: Model the roller coaster car as a particle undergoing uniform circular motion along a loop.

Visualize:



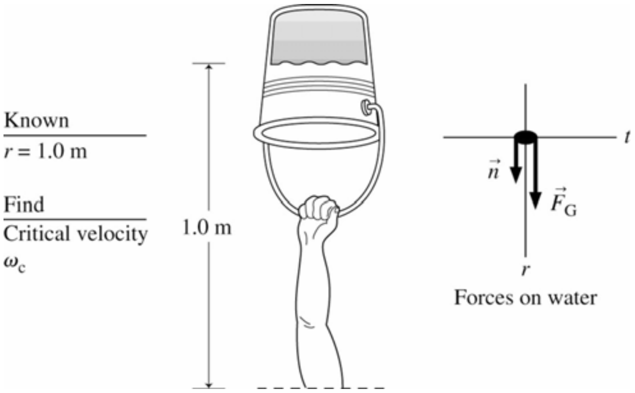
Notice that the r-axis points downward, toward the center of the circle.

Solve: In this problem the normal force is equal to the gravitational force: $n = F_G = mg$. We have

$$\sum F_r = n + F_G = \frac{mv^2}{r} = mg + mg \Rightarrow v = \sqrt{2rg} = \sqrt{2(20 \text{ m})(9.8 \text{ m/s}^2)} = 19.8 \text{ m/s} \approx 20 \text{ m/s}$$

8.22. Model: Model the bucket of water as a particle in uniform circular motion.

Visualize:

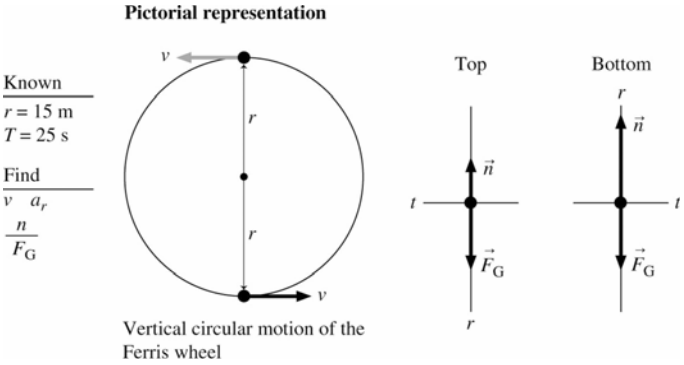


Solve: Let us say the distance from the bucket handle to the top of the water in the bucket is 35 cm. This makes the shoulder to water distance $65 \text{ cm} + 35 \text{ cm} = 1.00 \text{ m}$. The minimum angular velocity for swinging a bucket of water in a vertical circle without spilling any water corresponds to the case when the speed of the bucket is critical. In this case, $n = 0$ N when the bucket is in the top position of the circular motion. We get

$$\sum F_r = n + F_G = 0 \text{ N} + mg = \frac{m{v_c}^2}{r} = mr\omega_c^2$$

$$\Rightarrow \omega_c = \sqrt{g/r} = \sqrt{\frac{9.8 \text{ m/s}^2}{1.00 \text{ m}}} = 3.13 \text{ rad/s} = 3.13 \text{ rad/s} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}} = 30 \text{ rpm}$$

8.23. Model: Use the particle model for yourself while in uniform circular motion.
Visualize:



Solve: **(a)** The speed and acceleration are

$$v = \frac{2\pi r}{T} = \frac{2\pi(15\text{ m})}{25\text{ s}} = 3.77\text{ m/s} \quad a_r = \frac{v^2}{r} = \frac{(3.77\text{ m/s})^2}{15\text{ m}} = 0.95\text{ m/s}^2$$

So the speed is 3.8 m/s and the centripetal acceleration is 0.95 m/s^2.

(b) The weight $w = m,$ the normal force. On the ground, your weight is the same as the gravitational force F_G. Newton's second law at the top is

$$\sum F_r = F_G - n = ma_r = \frac{mv^2}{r}$$

$$\Rightarrow n = w = m\left(g - \frac{v^2}{r}\right) = m\left(9.8\text{ m/s}^2 - \frac{(3.77\text{ m/s})^2}{15\text{ m}}\right) = m(8.85\text{ m/s}^2)$$

$$\Rightarrow \frac{w}{F_G} = \frac{8.85\text{ m/s}^2}{9.8\text{ m/s}^2} = 0.90$$

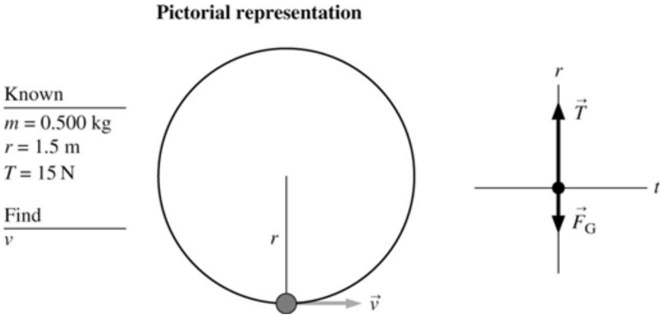
(c) Newton's second law at the bottom is

$$\sum F_r = n - F_G = ma_r = \frac{mv^2}{r}$$

$$\Rightarrow n = w = m\left(g + \frac{v^2}{r}\right) = m\left(9.8\text{ m/s}^2 + \frac{(3.77\text{ m/s}^2)}{15\text{ m}}\right) = (10.75\text{ m/s}^2)m$$

$$\Rightarrow \frac{w}{F_G} = \frac{10.75\text{ m/s}^2}{9.8\text{ m/s}^2} = 1.1$$

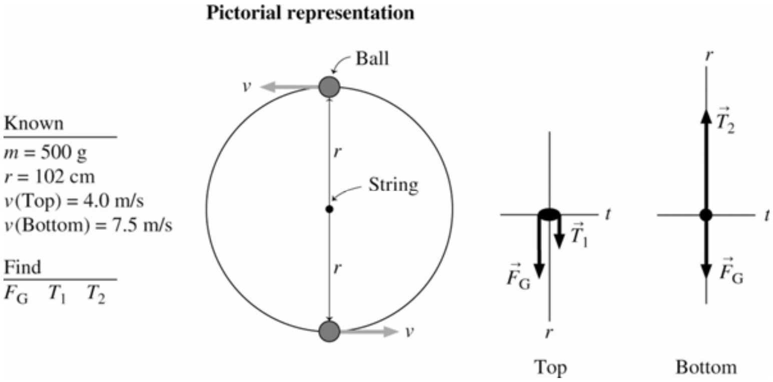
8.24. Model: Model the ball as a particle which is in a vertical circular motion.
Visualize:



Solve: At the bottom of the circle,

$$\sum F_r = T - F_G = \frac{mv^2}{r} \Rightarrow (15\text{ N}) - (0.500\text{ kg})(9.8\text{ m/s}^2) = \frac{(0.500\text{ kg})v^2}{(1.5\text{ m})} \Rightarrow v = 5.5\text{ m/s}$$

8.25. Model: Model the ball as a particle that is moving in a vertical circle.
Visualize:



Solve: (a) The ball's gravitational force $F_G = mg = (0.500\text{ kg})(9.8\text{ m/s}^2) = 4.9\text{ N}$.
(b) Newton's second law at the top is

$$\sum F_r = T_1 + F_G = ma_r = m\frac{v^2}{r}$$

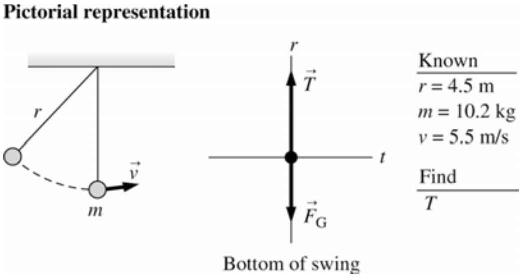
$$\Rightarrow T_1 = m\left(\frac{v^2}{r} - g\right) = (0.500\text{ kg})\left[\frac{(4.0\text{ m/s})^2}{1.02\text{ m}} - 9.8\text{ m/s}^2\right] = 2.9\text{ N}$$

(c) Newton's second law at the bottom is

$$\sum F_r = T_2 - F_G = \frac{mv^2}{r}$$

$$\Rightarrow T_2 = m\left(g + \frac{v^2}{r}\right) = (0.500\text{ kg})\left[9.8\text{ m/s}^2 + \frac{(7.5\text{ m/s})^2}{1.02\text{ m}}\right] = 32\text{ N}$$

8.26. Model: The ball is a particle on a massless rope in circular motion about the point where the rope is attached to the ceiling.
Visualize:



Solve: Newton's second law in the radial direction is

$$(\sum F_r) = T - F_G = T - mg = \frac{mv^2}{r}$$

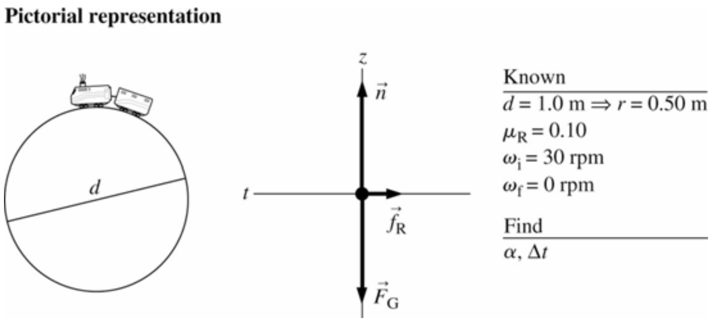
Solving for the tension in the rope and evaluating,

$$T = m\left(g + \frac{v^2}{r}\right) = (10.2\ \text{kg})\left(9.8\ \text{m/s}^2 + \frac{(5.5\ \text{m/s})^2}{4.5\ \text{m}}\right) = 168\ \text{N}$$

Assess: The tension in the rope is greater than the gravitational force on the ball in order to keep the ball moving in a circle.

Section 8.5 Nonuniform Circular Motion

8.27. Model: The train is a particle undergoing nonuniform circular motion.
Visualize:



Solve: First find the angular acceleration. Newton's second law in the vertical direction is

$$(F_{\text{net}})_y = n - F_G = 0$$

from which $n = mg$. The rolling friction is $f_R = \mu_R n = \mu_R mg$. This force provides the tangential acceleration

$$a_t = -\frac{f_R}{m} = -\mu_R g$$

The angular acceleration is

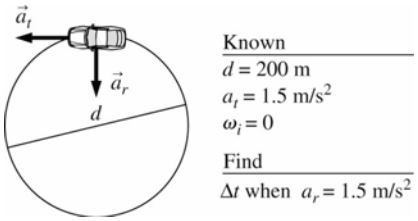
$$\alpha = \frac{a_t}{r} = \frac{-\mu_R g}{r} = \frac{-(0.10)(9.8\ \text{m/s}^2)}{0.50\ \text{m}} = -1.96\ \text{rad/s}^2$$

Use the angular acceleration and the change in angular velocity to find the time interval. The initial angular velocity is $30\left(\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right) = 3.14\ \text{rad/s}$. The time to come to a stop due to the rolling friction is

$$\Delta t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - 3.14\ \text{rad/s}}{-1.96\ \text{rad/s}^2} = 1.6\ \text{s}$$

Assess: The original angular speed of π rad/s means the train goes around the track one time every 2 seconds, so a stopping time of less than 2 s is reasonable.

8.28. Model: Use the particle model for the car, which is undergoing nonuniform circular motion.
Visualize:



Solve: The car is in circular motion with radius $r = \dfrac{d}{2} = 100\text{ m}$. We require

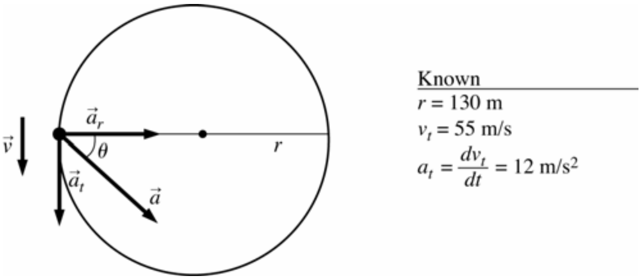
$$a_r = \omega^2 r = 1.5\text{ m/s}^2 \Rightarrow \omega = \sqrt{\frac{1.5\text{ m/s}^2}{r}} = \sqrt{\frac{1.5\text{ m/s}^2}{100\text{ m}}} = 0.122\text{ s}^{-1}$$

The definition of the angular velocity can be used to determine the time Δt using the angular acceleration $\alpha = \dfrac{a_t}{r} = \dfrac{1.5\text{ m/s}^2}{100\text{ m}} = 1.5\times10^{-2}\text{ s}^{-2}$.

$$\omega = \omega_\text{i} + \alpha\Delta t$$

$$\Rightarrow \Delta t = \frac{\omega - \omega_\text{i}}{\alpha} = \frac{0.122\text{ s}^{-1} - 0\text{ s}^{-1}}{0.015\text{ s}^{-2}} = 8.2\text{ s}$$

8.29. Model: The plane is a particle undergoing nonuniform circular motion. Make the *r*-*t* plane vertical; then there will be on motion in the z-direction. Neglect air resistance.
Visualize:



Solve: We are given the tangential acceleration and we find the centripetal acceleration from $a_r = v^2/r$. Use the second law to find the components of the force.

$$F_r = m\frac{v^2}{r} = (85000\text{ kg})\frac{(55\text{ m/s})^2}{130\text{ m}} = 1.978\times10^6\text{ N}$$

$$F_t = ma_t = (85000\text{ kg})(12\text{ m/s}^2) = 1.02\times10^6\text{ N}$$

(a) Now find the magnitude of the net force.

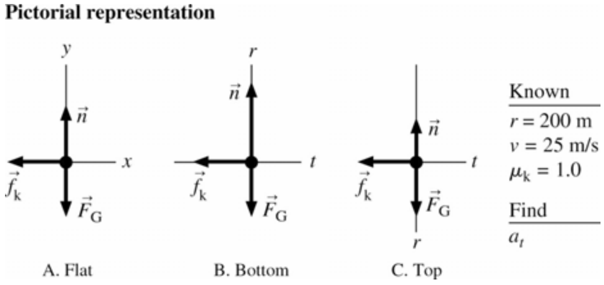
$$F_\text{net} = \sqrt{F_r^2 + F_t^2} = \sqrt{(1.978\times10^6\text{ N})^2 + (1.02\times10^6\text{ N})^2} = 2.2\times10^6\text{ N}$$

(b) Find the angle the net force makes with the horizontal.

$$\theta = \tan^{-1}\frac{F_t}{F_r} = \tan^{-1}\frac{-1.02\times10^6\text{ N}}{1.978\times10^6\text{ N}} = -27°$$

The angle is negative because it is below the horizontal.
Assess: The plane's velocity is down and increasing, so the net force must be below the horizontal.

8.30. Model: Use the particle model for the car and the model of kinetic friction.
Visualize:



Solve: We will apply Newton's second law to all three cars.
Car A:

$$\sum F_x = n_x + (f_\text{k})_x + (F_\text{G})_x = 0\text{ N} - f_\text{k} + 0\text{ N} = ma_x$$

$$\sum F_y = n_y + (f_\text{k})_y + y_y = n + 0\text{ N} - mg = 0\text{ N}$$

The y-component equation means $n = mg$. Since $f_\text{k} = \mu_\text{k} n$, we have $f_\text{k} = \mu_\text{k} mg$. From the x-component equation,

$$a_x = \frac{-f_\text{k}}{m} = \frac{-\mu_\text{k} mg}{m} = -\mu_\text{k}\text{g} = -9.8\text{ m/s}^2$$

Car B: Car B is in circular motion with the center of the circle above the car.

$$\sum F_r = n_r + (f_\text{k})_r + (F_\text{G})_r = n + 0\text{ N} - mg = ma_r = \frac{mv^2}{r}$$

$$\sum F_t = n_t + (f_\text{k})_t + (F_\text{G})_t = 0\text{ N} - f_\text{k} + 0\text{ N} = +ma_t$$

From the r-equation

$$n = mg + \frac{mv^2}{r} \Rightarrow f_\text{k} = \mu_\text{k} n = \mu_\text{k} m\left(g + \frac{v^2}{r}\right)$$

Substituting back into the t-equation,

$$a_t = -\frac{f_\text{k}}{m} = -\frac{\mu_\text{k} m}{m}\left(g + \frac{v^2}{r}\right) = -\mu_\text{k}\left(9.8\text{ m/s}^2 + \frac{(25\text{ m/s})^2}{200\text{ m}}\right) = -12.9\text{ m/s}^2$$

Car C: Car C is in circular motion with the center of the circle below the car.

$$\sum F_r = n_r + (f_\text{k})_r + (F_\text{G})_r = -n + 0\text{ N} + mg = ma_r = \frac{mv^2}{r}$$

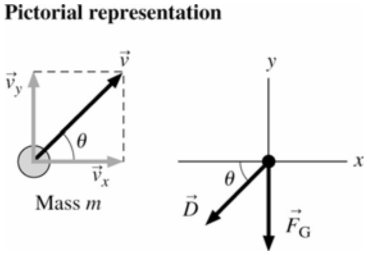
$$\sum F_t = n_t + (f_\text{k})_t + (F_\text{G})_t = 0\text{ N} - f_\text{k} + 0\text{ N} = ma_t$$

From the r-equation $n = m(g - v^2/r)$. Substituting this into the t-equation yields

$$a_t = \frac{-f_\text{k}}{m} = \frac{-\mu_\text{k} n}{m} = -\mu_\text{k}(g - v^2/r) = -6.7\text{ m/s}^2$$

Problems

8.31. Visualize:



Solve: From Chapter 6 the drag on a projectile is $\vec{D} = \left(\frac{1}{4}Av^2, \text{ direction opposite to motion}\right)$, where A is the cross-sectional area. Using the free-body diagram above, apply Newton's second law to each direction.

In the *x*-direction,

$$a_x = \frac{(F_\text{net})_x}{m} = -\frac{D\cos\theta}{n} = -\frac{\frac{1}{4}Av^2\cos\theta}{m}$$

$$a_y = \frac{(F_\text{net})_y}{m} = -\frac{D\sin\theta - F_\text{G}}{m} = -\frac{\frac{1}{4}Av^2\sin\theta}{m} - g$$

Since $v = \sqrt{v_x^2 + v_y^2}$, $v_x = v\cos\theta$, and $v_y = v\sin\theta$, we can rewrite these as

$$a_x = -\frac{A(v\cos\theta)v}{4\text{ m}} = -\frac{Av_x\sqrt{v_x^2 + v_y^2}}{4\text{ m}}$$

$$a_y = -\frac{A(v\sin\theta)v}{4\text{ m}} - g = -g - \frac{Av_y\sqrt{v_x^2 + v_y^2}}{4\text{ m}}$$

8.32. Model: The plane is a particle undergoing nonuniform circular motion. Make the *r*-t plane vertical; then there will be on motion in the z-direction. Neglect air resistance.
Visualize: The wire is a parabola opening up with the vertex at the origin. We are given $m = 0.10$ kg.

Solve: (a) Use the basic calculus definitions of *a* and *v* and use $y = (2\text{ m}^{-1})x^2$. First use the chain rule to find an expression for v_y.

$$v_y = \frac{dy}{dt} = \frac{d((2\text{ m}^{-1})x^2)}{dt} = (4\text{ m}^{-1})x\frac{dx}{dt} = (4\text{ m}^{-1})xv_x$$

Use that result in the expression for a_y. Use the product rule.

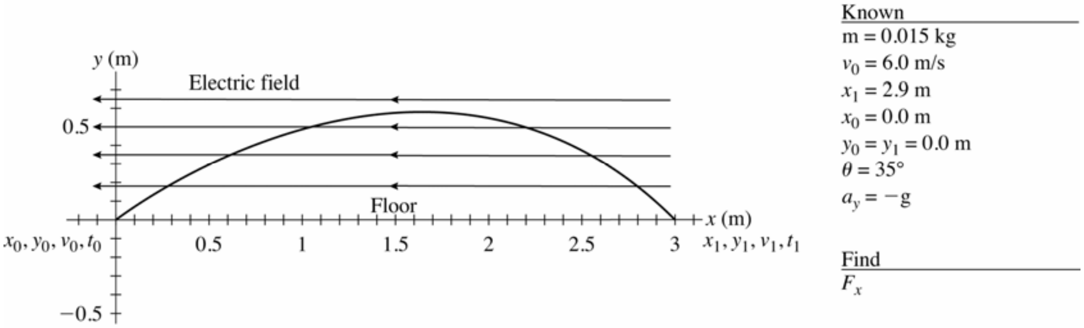
$$a_y = \frac{dv_y}{dt} = \frac{d(4xv_x)}{dt} = (4\text{ m}^{-1})\left[x\frac{dv_x}{dt} + v_x\frac{dx}{dt}\right] = (4\text{ m}^{-1})(xa_x + v_x^2)$$

(b) At the instant the bead is at the lowest point it is not accelerating in the *x*-direction; the net force is in the *y*-direction.

$$F_\text{net} = F_y = ma_y = (m)(4\text{ m}^{-1})(xa_x + v_x^2) = (0.10\text{ kg})(4\text{ m}^{-1})(0 + (2.3\text{ m/s})^2) = (2.1\text{ N})\,\hat{j}$$

Assess: We have carefully tracked the units, and they work out.

8.33. Model: The object feels constant forces in both the horizontal and vertical directions.
Visualize: The object's trajectory is not a parabola because neither component has a constant velocity.



Known
$m = 0.015$ kg
$v_0 = 6.0$ m/s
$x_1 = 2.9$ m
$x_0 = 0.0$ m
$y_0 = y_1 = 0.0$ m
$\theta = 35°$
$a_y = -g$

Find
F_x

Solve: Use the kinematic equations in each direction. Find the time the trajectory takes from the *y* equation.

$$y_1 = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \Rightarrow v_0\sin\theta = \tfrac{1}{2}gt \Rightarrow t = \frac{2v_0\sin\theta}{g} = \frac{2(6.0\text{ m/s})\sin 35°}{9.8\text{ m/s}^2} = 0.702\text{ s}$$

Use this t in the x-equation to find a_x.

$$x_1 = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \Rightarrow a_x = 2\frac{x_1 - v_0\cos\theta t}{t^2} = 2\frac{(2.9\text{ m}) - (6.0\text{ m})\cos 35°(0.702\text{ s})}{(0.702\text{ s})^2} = -2.24\text{ m/s}^2$$

Now use the second law to find the force. Use the absolute value of the acceleration since we seek the magnitude only.

$$F_e = ma_x = (0.015\text{ kg})(2.24\text{ m/s}^2) = 0.034\text{ N}$$

Assess: This seems like a reasonable force on a small object.

8.34. Model: Use the particle model and the constant-acceleration equations of kinematics for the rocket.

Solve: (a) The acceleration of the rocket in the launch direction is obtained from Newton's second law $\vec{F} = m\vec{a}$:

$$140{,}700\text{ N} = (5000\text{ kg})a \Rightarrow a = 28.14\text{ m/s}^2$$

Therefore, $a_x = a\cos 44.7° = 20.0\text{ m/s}^2$ and $a_y = a\sin 44.7° = 19.8\text{ m/s}^2$. The net acceleration in the y-direction is thus

$$(a_\text{net})_y = a_y - g = (19.8 - 9.8)\text{ m/s}^2 = 10.0\text{ m/s}^2$$

With this acceleration, we can write the equations for the x- and y-motions of the rocket.

$$y = y_0 + v_{0y}(t - t_0) + \tfrac{1}{2}(a_\text{net})_y(t - t_0)^2 = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(10.0\text{ m/s}^2)t^2 = (5.00\text{ m/s}^2)t^2$$

$$x = x_0 + v_{0x}(t - t_0) + \tfrac{1}{2}(a_\text{net})_x(t - t_0)^2 = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(20.0\text{ m/s}^2)t^2 = (10.0\text{ m/s}^2)t^2$$

From these two equations,

$$\frac{x}{y} = \frac{(10.0\text{ m/s}^2)t^2}{(5.00\text{ m/s}^2)t^2} = 2$$

The equation that describes the rocket's trajectory is $y = \tfrac{1}{2}x$.

(b) It is a straight line with a slope of $\tfrac{1}{2}$.

(c) In general,

$$v_y = v_{0y} + (a_\text{net})_y(t_1 - t_0) = 0 + (10.0\text{ m/s}^2)t_1$$

$$v_x = v_{0x} + (a_\text{net})_x(t_1 - t_0) = 0 + (20.0\text{ m/s}^2)t_1$$

$$v = \sqrt{(10.0\text{ m/s}^2)^2 t_1^2 + (20.0\text{ m/s}^2)^2 t_1^2} = (22.36\text{ m/s}^2)t_1$$

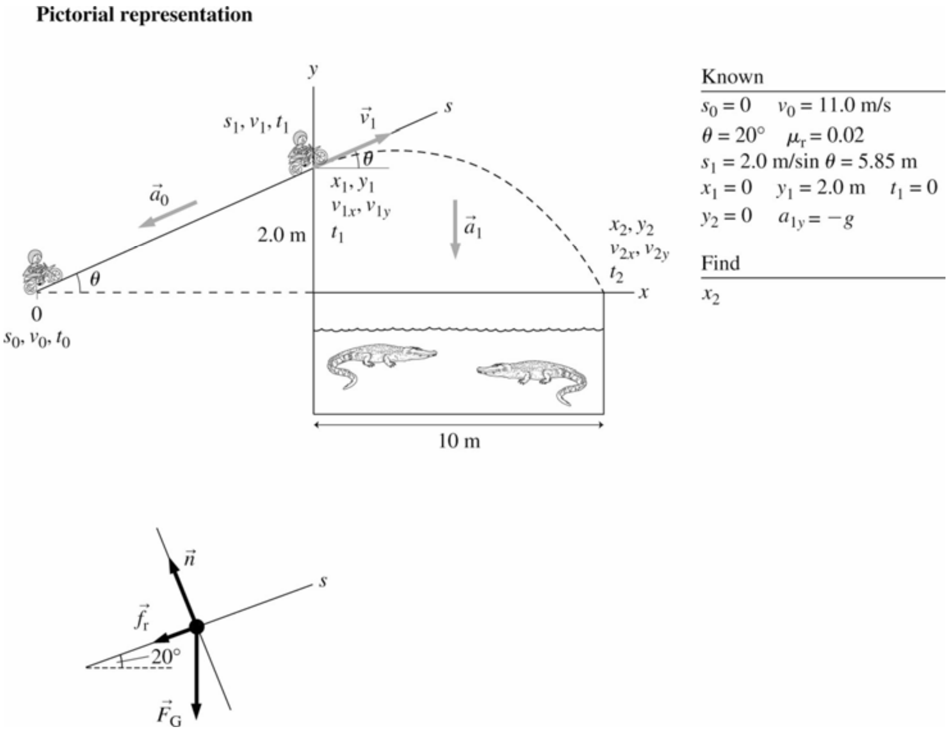
The time required to reach the speed of sound is calculated as follows:

$$330\text{ m/s} = (22.36\text{ m/s}^2)t_1 \Rightarrow t_1 = 14.76\text{ s}$$

We can now obtain the elevation of the rocket. From the y-equation,

$$y = (5.00\text{ m/s}^2)t_1^2 = (5.00\text{ m/s}^2)(14.76\text{ s})^2 = 1090\text{ m}$$

8.35. Model: Treat the motorcycle and rider as a particle.
Visualize: This is a two-part problem. Use an *s*-axis parallel to the slope for the first part, regular *xy*-coordinates for the second. The motorcycle's final velocity at the top of the ramp is its initial velocity as it becomes airborne.



Solve: The motorcycle's acceleration on the ramp is given by Newton's second law:

$$(F_{\text{net}})_s = -f_r - mg\sin 20° = -\mu_r n - mg\sin 20° = -\mu_r mg\cos 20° - mg\sin 20° = ma_0$$
$$a_0 = -g(\mu_r\cos 20° + \sin 20°) = -(9.8\text{ m/s}^2)((0.02)\cos 20° + \sin 20°) = -3.536\text{ m/s}^2$$

The length of the ramp is $s_1 = (2.0\text{ m})/\sin 20° = 5.85\text{ m}$. We can use kinematics to find its speed at the top of the ramp:

$$v_1^2 = v_0^2 + 2a_0(s_1 - s_0) = v_0^2 + 2a_0 s_1$$
$$\Rightarrow v_1 = \sqrt{(11.0\text{ m/s})^2 + 2(-3.536\text{ m/s}^2)(5.85\text{ m})} = 8.92\text{ m/s}$$

This is the motorcycle's initial speed into the air, with velocity components $v_{1x} = v_1\cos 20° = 8.38\text{ m/s}$ and $v_{1y} = v_1\sin 20° = 3.05\text{ m/s}$. We can use the *y*-equation of projectile motion to find the time in the air:

$$y_2 = 0\text{ m} = y_1 + v_{1y}t_2 + \tfrac{1}{2}a_{1y}t_2^2 = 2.0\text{ m} + (3.05\text{ m/s})t_2 - (4.90\text{ m/s}^2)t_2^2$$

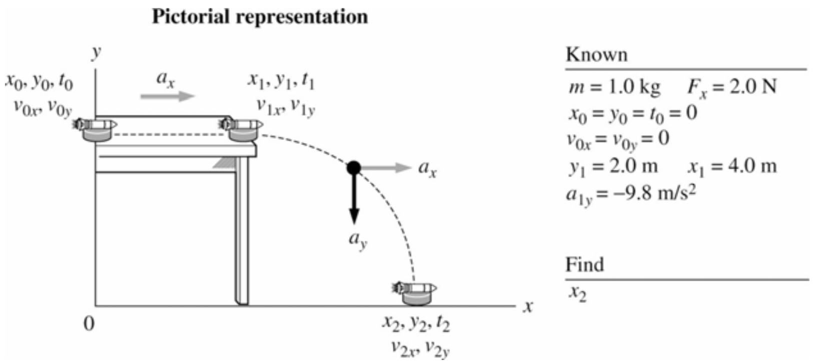
This quadratic equation has roots $t_2 = -0.399\text{ s}$ (unphysical) and $t_2 = 1.021\text{ s}$. The *x*-equation of motion is thus

$$x_2 = x_1 + v_{1x}t_2 = 0\text{ m} + (8.38\text{ m/s})t_2 = 8.6\text{ m}$$

$8.56\text{ m} < 10.0\text{ m}$, so it looks like crocodile food.

8.36. Model: The hockey puck will be treated as a particle whose motion is determined by constant-acceleration kinematic equations. We break this problem in two parts, the first pertaining to motion on the table and the second to free fall.

Visualize:



Solve: Newton's second law is:

$$F_x = ma_x \Rightarrow a_x = \frac{F_x}{m} = \frac{2.0 \text{ N}}{1.0 \text{ kg}} = 2.0 \text{ m/s}^2$$

The kinematic equation $v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0)$ yields:

$$v_{1x}^2 = 0 \text{ m}^2/\text{s}^2 + 2(2.0 \text{ m/s}^2)(4.0 \text{ m}) \Rightarrow v_{1x} = 4.0 \text{ m/s}$$

Let us now find the time of free fall $(t_2 - t_1)$:

$$y_2 = y_1 + v_{1y}(t_2 - t_1) + \tfrac{1}{2}a_{1y}(t_2 - t_1)^2$$

$$\Rightarrow 0 \text{ m} = 2.0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(t_2 - t_1)^2 \Rightarrow (t_2 - t_1) = 0.639 \text{ s}$$

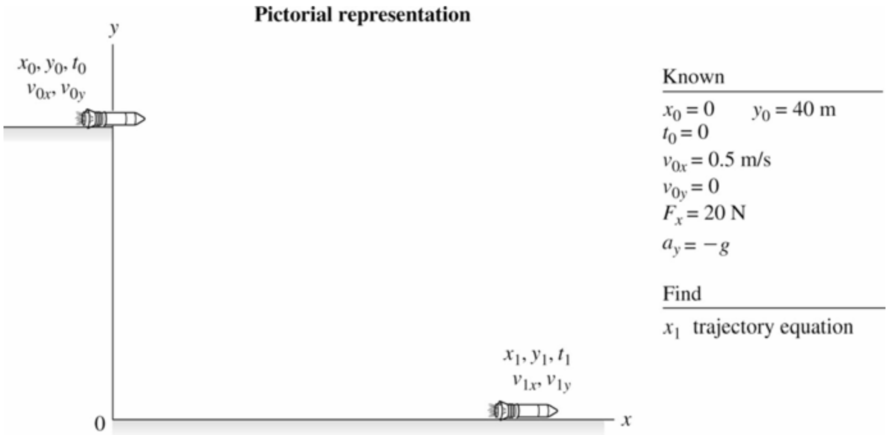
Having obtained v_{1x} and $(t_2 - t_1)$, we can now find $(x_2 - x_1)$ as follows:

$$x_2 = x_1 + v_{1x}(t_2 - t_1) + \tfrac{1}{2}a_x(t_2 - t_1)^2$$

$$\Rightarrow x_2 - x_1 = (4.0 \text{ m/s})(0.639 \text{ s}) + \tfrac{1}{2}(2.0 \text{ m/s}^2)(0.639 \text{ s})^2 = 3.0 \text{ m}$$

Assess: For a modest horizontal thrust of 2.0 N, a landing distance of 3.0 m is reasonable.

8.37. Model: The model rocket is treated as a particle and its motion is determined by constant-acceleration kinematic equations.
Visualize:



Solve: As the rocket is accidentally bumped $v_{0x} = 0.5\text{ m/s}$ and $v_{0y} = 0\text{ m/s}$. On the other hand, when the engine is fired

$$F_x = ma_x \Rightarrow a_x = \frac{F_x}{m} = \frac{20\text{ N}}{0.500\text{ kg}} = 40\text{ m/s}^2$$

(a) Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$0\text{ m} = 40\text{ m} + 0\text{ m} + \tfrac{1}{2}(-9.8\text{ m/s}^2)t_1^2 \Rightarrow t_1 = 2.857\text{ s}$$

The distance from the base of the wall is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 = 0\text{ m} + (0.5\text{ m/s})(2.857\text{ s}) + \tfrac{1}{2}(40\text{ m/s}^2)(2.857\text{ s})^2 = 165\text{ m}$$

(b) The *x*- and *y*-equations are

$$y = y_0 + v_{0y}(t - t_0) + \tfrac{1}{2}a_y(t - t_0)^2 = 40 - 4.9t^2$$

$$x = x_0 + v_{0x}(t - t_0) + \tfrac{1}{2}a_x(t - t_0)^2 = 0.5t + 20t^2$$

Except for a brief interval near $t = 0$, $20t^2 \gg 0.5t$. Thus $x \approx 20t^2$, or $t^2 = x/20$. Substituting this into the *y*-equation gives

$$y = 40 - 0.245x$$

This is the equation of a straight line, so the rocket follows a linear trajectory to the ground.

8.38. Model: Treat the projectile as a particle, but the acceleration is not constant.

Visualize: Use $v_\text{f} = v_\text{i} + \int_{t_\text{i}}^{t_\text{f}} a\,dt$ and $a = F/m$ in each direction. We are given $m = 2.0\text{ kg}$, $t_\text{f} = 2.0\text{ s}$, and $\vec{v}_\text{i} = 8.0\hat{i}\text{ m/s}$.

Solve: (a) Analyze each direction separately:

$$v_{1x} = v_{0x} + \int_{t_0}^{t_1} \frac{F_x}{m}dt = 8.0\text{ m/s} + \frac{1}{2.0\text{ kg}}\int_0^2 (-2.0t)dt = 8.0\text{ m/s} + \frac{1}{2.0\text{ kg}}\left[-t^2\right]_0^2 = 6.0\text{ m/s}$$

$$v_{1y} = v_{0y} + \int_{t_0}^{t_1} \frac{F_y}{m}dt = 0\text{ m/s} + \frac{1}{2.0\text{ kg}}\int_0^2 (4.0t^2)dt = \frac{1}{2.0\text{ kg}}\left[\frac{4}{3}t^3\right]_0^2 = 5.33\text{ m/s}$$

Now find the magnitude of the velocity vector at 2.0 s.

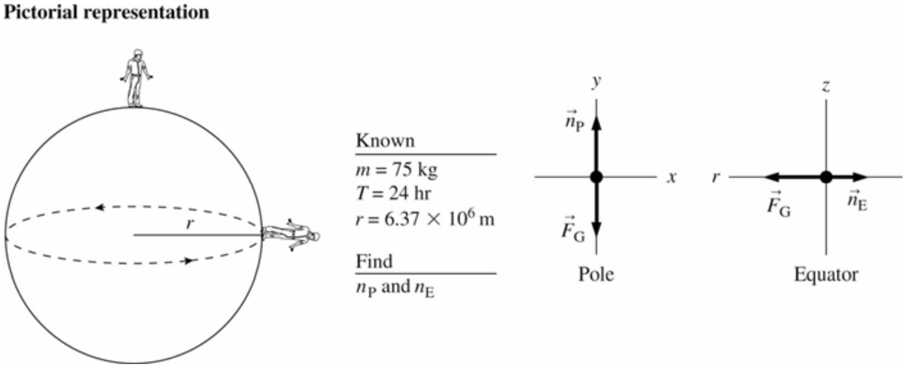
$$v_1 = \sqrt{(v_{1x})^2 + (v_{1y})^2} = \sqrt{(6.0\text{ m/s})^2 + (5.33\text{ m/s})^2} = 8.0\text{ m/s}$$

(b) The projectile will be moving parallel to the *y*-axis when the *x* component of the velocity is zero.

$$v_x = v_{0x} + \frac{1}{m}\int_0^t F_x dt = 0 \Rightarrow 8.0\text{ m/s} + \frac{1}{2.0\text{ kg}}\left[-t^2\right]_0^t = 0 \Rightarrow 8.0\text{ m/s} = \frac{1}{2.0\text{ kg}}t^2 \Rightarrow t = 4.0\text{ s}$$

Assess: The speed after 2.0 s is approximately the same as at the beginning, but it is in a different direction. Given the data, 4.0 s in part **(b)** seems reasonable.

8.39. Model: Treat the man as a particle. The man at the equator undergoes uniform circular motion as the earth rotates.
Visualize:



Solve: The scale reads the man's weight $F_G = n$, the force of the scale pushing up against his feet. At the north pole, where the man is in static equilibrium,

$$n_P = F_G = mg = 735 \text{ N}$$

At the equator, there must be a net force toward the center of the earth to keep the man moving in a circle. The *r*-axis points toward the center, so

$$\sum F_r = F_G - n_E = m\omega^2 r \Rightarrow n_E = mg - m\omega^2 r = n_P - m\omega^2 r$$

The equator scale reads less than the north pole scale by the amount $m\omega^2 r$. The man's angular velocity is that of the equator, or

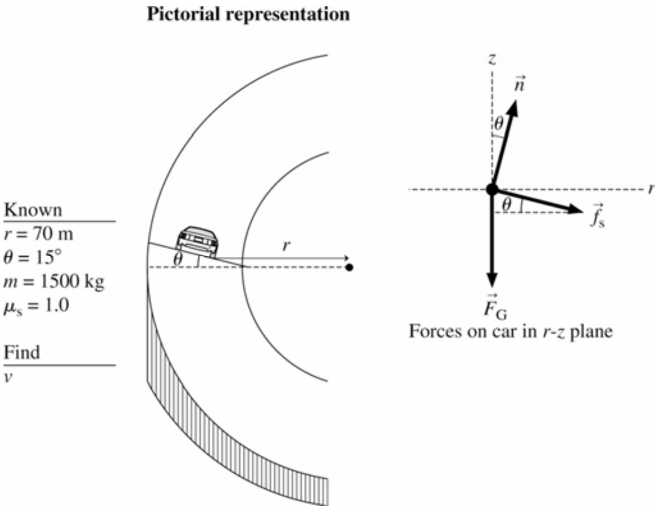
$$\omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{24 \text{ hours} \times (3600 \text{ s/1 h})} = 7.27 \times 10^{-5} \text{ rad/s}$$

Thus the north pole scale reads more than the equator scale by

$$\Delta w = (75 \text{ kg})(7.27 \times 10^{-5} \text{ rad/s})^2 (6.37 \times 10^6 \text{ m}) = 2.5 \text{ N}$$

Assess: The man at the equator appears to have lost $\Delta m = \Delta w/g \approx 0.25$ kg, or the equivalent of $\approx \frac{1}{2}$ lb.

8.40. Model: We will use the particle model for the car, which is undergoing uniform circular motion on a banked highway, and the model of static friction.
Visualize:



Note that we need to use the coefficient of static friction μ_s, which is 1.0 for rubber on concrete.

Solve: Newton's second law for the car is

$$\sum F_r = f_s\cos\theta + n\sin\theta = \frac{mv^2}{r} \quad \sum F_z = n\cos\theta - f_s\sin\theta - F_G = 0\text{ N}$$

Maximum speed is when the static friction force reaches its maximum value $(f_s)_{max} = \mu_s n$. Then

$$n(\mu_s\cos15° + \sin15°) = \frac{mv^2}{r} \quad n(\cos15° - \mu_s\sin15°) = mg$$

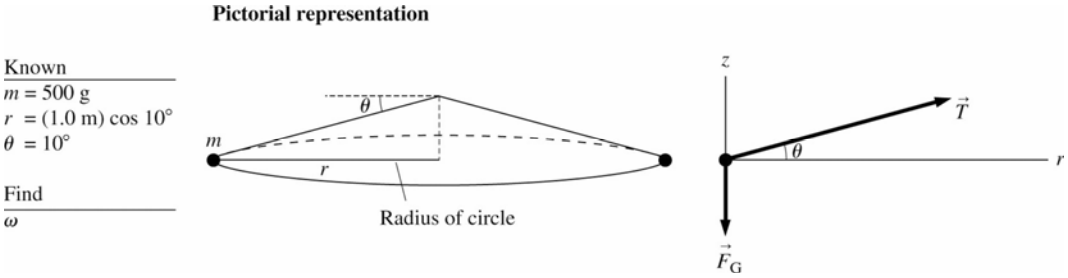
Dividing these two equations and simplifying, we get

$$\frac{\mu_s + \tan15°}{1 - \mu_s\tan15°} = \frac{v^2}{gr} \Rightarrow v = \sqrt{gr\frac{\mu_s + \tan15°}{1 - \mu_s\tan15°}}$$

$$= \sqrt{(9.80\text{ m/s}^2)(70\text{ m})\frac{(1.0 + 0.268)}{(1 - 0.268)}} = 34\text{ m/s}$$

Assess: The above value of $34\text{ m/s} \approx 70\text{ mph}$ is reasonable.

8.41. Model: Use the particle model for the rock, which is undergoing uniform circular motion.
Visualize: L is the hypotenuse of the right triangle. The radius of the circular motion is $r = L\cos\theta$.



Solve:
(a) Apply Newton's second law in the z- and r-directions.

$$\sum F_z = T\sin\theta - mg = 0 \Rightarrow T = \frac{mg}{\sin\theta}$$

$$\sum F_r = T\cos\theta = m\omega^2 r = m\omega^2(L\cos\theta) \Rightarrow T = m\omega^2 L$$

Set the two expressions for T equal to each other and solve for ω.

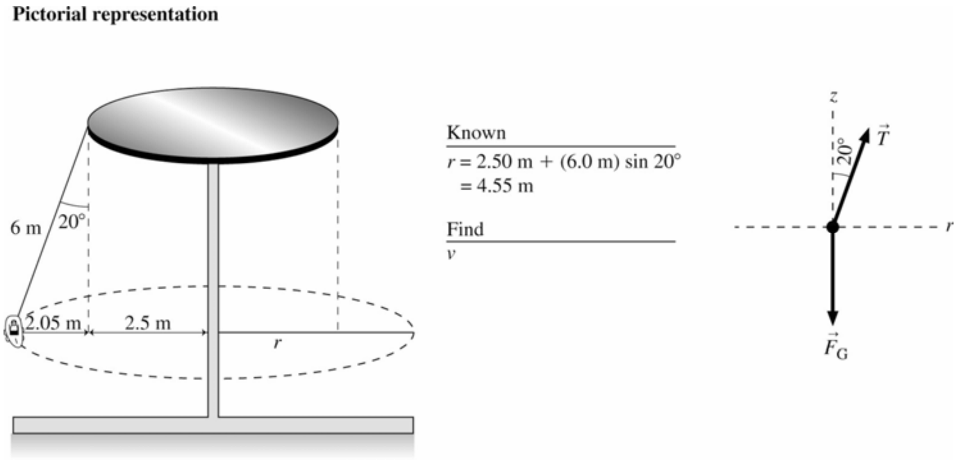
$$\frac{mg}{\sin\theta} = m\omega^2 L \Rightarrow \omega = \sqrt{\frac{g}{L\sin\theta}}$$

(b) Insert $L = 1.0\text{ m}$ and $\theta = 10°$.

$$\omega = \sqrt{\frac{g}{L\sin\theta}} = \sqrt{\frac{9.8\text{ m/s}^2}{(1.0\text{ m})\sin10°}} = 7.51\text{ rad/s}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 72\text{ rpm}$$

Assess: Notice that the mass canceled out of the equation so the 500 g was unnecessary information. In other words, the answer, 72 rpm, would be the same regardless of the mass.
The dependencies of ω on g, L, and θ seem to be in the right directions.

8.42. Model: Use the particle model for the car, which is in uniform circular motion.
Visualize:



Solve: Newton's second law is

$$\sum F_r = T\sin 20° = ma_r = \frac{mv^2}{r} \quad \sum F_z = T\cos 20° - F_G = 0 \text{ N}$$

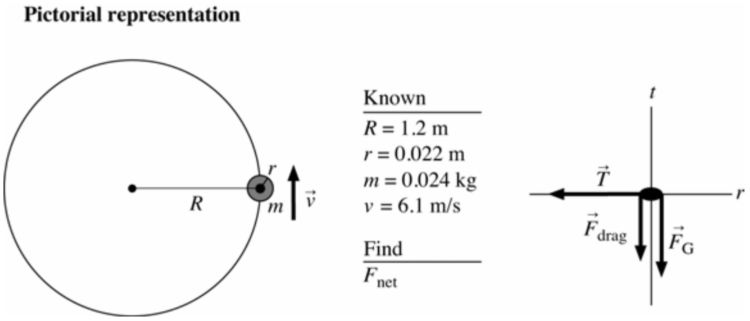
These equations can be written as

$$T\sin 20° = \frac{mv^2}{r} \quad T\cos 20° = mg$$

Dividing these two equations gives

$$\tan 20° = v^2/rg \Rightarrow v = \sqrt{rg\tan 20°} = \sqrt{(4.55 \text{ m})(9.8 \text{ m/s}^2)\tan 20°} = 4.03 \text{ m/s} \approx 4 \text{ m/s}$$

8.43. Model: The ball is in nonuniform circular motion.
Visualize: Use *r* for the radius of the ball, and *R* for the radius of the circular path.



Solve: Find the force in the *r* and *t* directions.

$$F_r = ma_r = m\frac{v^2}{R} = (0.024 \text{ kg})\frac{(6.1 \text{ m/s})^2}{(1.2 \text{ m})} = 0.744 \text{ N}$$

$$F_t = -F_{\text{drag}} - mg = -\left(\tfrac{1}{2}(0.5)(1.2 \text{ kg/m}^2)\pi(0.022 \text{ m})^2(6.1 \text{ m/s})^2\right) - (0.024 \text{ kg})(9.8 \text{ m/s}^2) = -0.252 \text{ N}$$

$$F_{\text{net}} = \sqrt{F_r^2 + F_t^2} = \sqrt{(0.744 \text{ N})^2 + (-.252 \text{ N})^2} = 0.79 \text{ N}$$

The tangential (downward) component is quite negligible compared to the radial component.

Assess: The radial component of the force is quite large; it must be supplied by the string.

8.44. Model: The particle is in uniform circular motion because we neglect gravity.

Visualize: Given $F = qvB$ is the net force and is centripetally directed, we also have $a = v^2/r$.

Solve: Set $qvB = mv^2/r$ and solve for *v*.

$$F_{\text{net}} = ma = m\frac{v^2}{r} = qvB \Rightarrow v = \frac{qBr}{m}$$

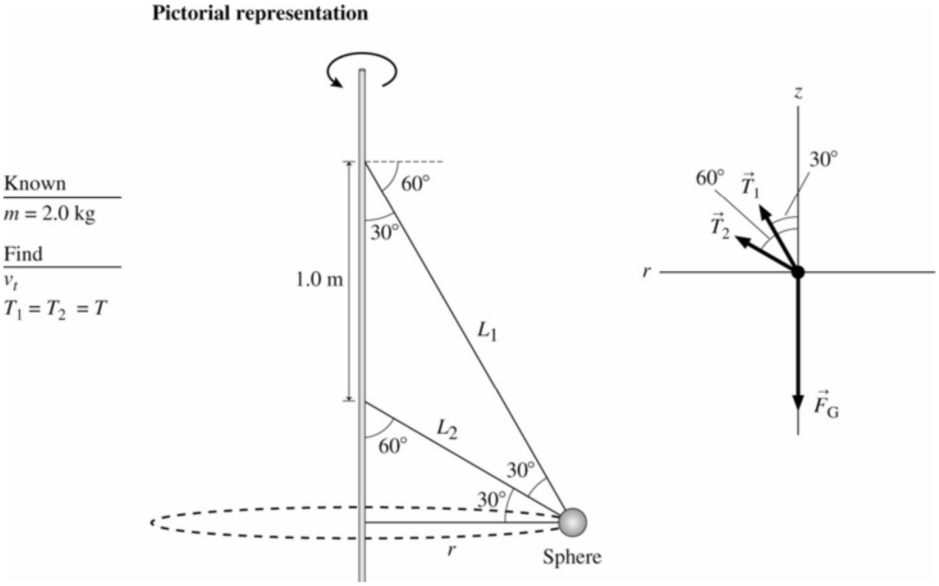
Now find the period *T* from the circumference and speed.

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\left(\dfrac{qBr}{m}\right)} = \frac{2\pi m}{qB}$$

Assess: The units work out correctly. It seems proper that the period would depend directly on the mass of the particle and inversely with the charge and magnetic field.

8.45. Model: Assume the particle model for a sphere in circular motion at constant speed.

Visualize:



Solve: (a) Newton's second law along the *r* and *z* axes is:

$$\sum F_r = T_1 \sin 30° + T_2 \sin 60° = \frac{mv_t^2}{r} \quad \sum F_z = T_1 \cos 30° + T_2 \cos 60° - F_G = 0 \text{ N}$$

Since we want $T_1 = T_2 = T$, these two equations become

$$T(\sin 30° + \sin 60°) = \frac{mv_t^2}{r} \quad T(\cos 30° + \cos 60°) = mg$$

Since $\sin 30° + \sin 60° = \cos 30° + \cos 60°$,

$$mg = \frac{mv_t^2}{r} \Rightarrow v_t = \sqrt{rg}$$

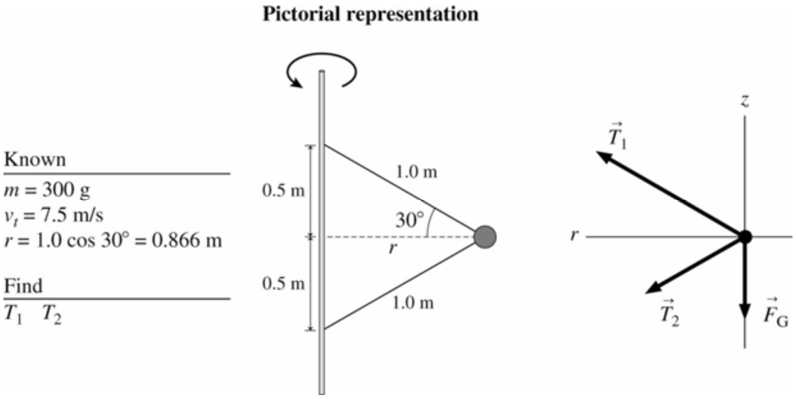
The triangle with sides L_1, L_2, and 1.0 m is isosceles, so $L_2 = 1.0$ m and $r = L_2 \cos 30°$. Thus

$$\sqrt{L_2 \cos 30° g} = \sqrt{(1.0 \text{ m}) \cos 30° g} = \sqrt{(0.866 \text{ m})(9.8 \text{ m/s}^2)} = 2.9 \text{ m/s}$$

(b) The tension is

$$T = \frac{mg}{\cos 30° + \cos 60°} = \frac{(2.0 \text{ kg})(9.8 \text{ m/s}^2)}{0.866 + 0.5} = 14.3 \text{ N} \approx 14 \text{ N}$$

8.46. Model: Use the particle model for a sphere revolving in a horizontal circle.
Visualize:



Solve: Newton's second law in the r- and z-directions is

$$\Sigma(F)_r = T_1 \cos 30° + T_2 \cos 30° = \frac{mv_t^2}{r} \qquad \Sigma(F)_z = T_1 \sin 30° - T_2 \sin 30° - F_G = 0 \text{ N}$$

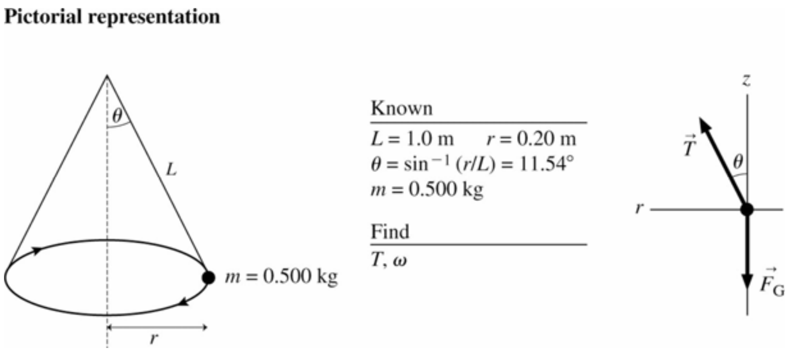
Using $r = (1.0 \text{ m})\cos 30° = 0.886 \text{ m}$, these equations become

$$T_1 + T_2 = \frac{mv_t^2}{r\cos 30°} = \frac{(0.300 \text{ kg})(7.5 \text{ m/s})^2}{(0.866 \text{ m})(0.866)} = 22.5 \text{ N}$$

$$T_1 - T_2 = \frac{mg}{\sin 30°} = \frac{(0.300 \text{ kg})(9.8 \text{ m/s}^2)}{(0.5)} = 5.88 \text{ N}$$

Solving for T_1 and T_2 yields $T_1 = 14.2 \text{ N} \approx 14 \text{ N}$ and $T_2 = 8.3 \text{ N}$.

8.47. Use the particle model for the ball, which is undergoing uniform circular motion.
Visualize: We are given L, r, and m, so our answers must be in terms of those variables. L is the hypotenuse of the right triangle. The ball moves in a *horizontal* circle of radius $r = L\cos\theta$. The acceleration and net force point toward the center of the circle, not along the string.



Solve:
(a) Apply Newton's second law in the z-direction.

$$\sum F_z = T\cos\theta - mg = 0 \Rightarrow T = \frac{mg}{\cos\theta}$$

From the right triangle $\cos\theta = \sqrt{L^2 - r^2}/L$.

$$T = \frac{mg}{\cos\theta} = \frac{mgL}{\sqrt{L^2 - r^2}}$$

(b) Apply Newton's second law in the r-direction.

$$\sum F_r = T\sin\theta = m\omega^2 r = m\omega^2(L\sin\theta) \Rightarrow T = m\omega^2 L.$$

Set the two expressions for T equal to each other, cancel m and one L, and solve for ω.

$$\frac{mgL}{\sqrt{L^2 - r^2}} = m\omega^2 L \Rightarrow \omega = \sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$$

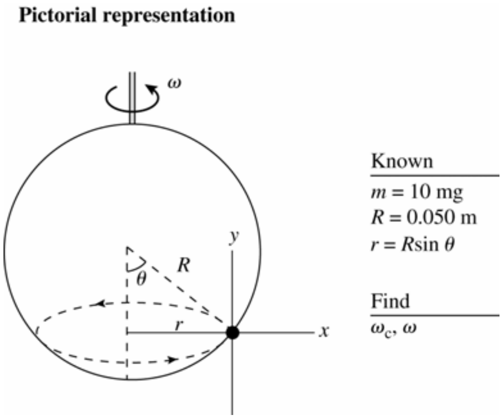
(c) Insert $L = 1.0\text{ m}$, $r = 0.20\text{ m}$ and $m = 0.50\text{ kg}$.

$$T = \frac{mgL}{\sqrt{L^2 - r^2}} = \frac{(0.50\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m})}{\sqrt{(1.0\text{ m})^2 - (0.20\text{ m})^2}} = 5.0\text{ N}$$

$$\omega = \sqrt{\frac{g}{\sqrt{L^2 - r^2}}} = \sqrt{\frac{9.8\text{ m/s}^2}{\sqrt{(1.0\text{ m})^2 - (0.20\text{ m})^2}}} = 3.163\text{ rad/s}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 30\text{ rpm}$$

Assess: Notice that the mass canceled out of the equation for ω, but not for T, so the 500 g was necessary information.

8.48. Model: The particle is in uniform circular motion of radius $r = R\sin\theta$ where R is the radius of the loop of wire.
Visualize: The wire provides no component of the force tangent to the wire because it is frictionless. The wire can only exert a force normally (radially inward toward the center of the wire loop).



Solve: Use the second law on the bead in the x and y directions. There is no acceleration in the y direction and the acceleration in the x direction is the centripetal acceleration $a = \omega^2 r$.

$$\Sigma F_x = F_{\text{wire}}\sin\theta = m\omega^2 r$$
$$\Sigma F_y = F_{\text{wire}}\cos\theta - mg = 0$$

Solve the *y* equation for F_{wire} and insert into the *x* equation along with $r = R\sin\theta$.

$$F_{\text{wire}} = \frac{mg}{\cos\theta} \quad \Rightarrow \quad \frac{mg}{\cos\theta}\sin\theta = m\omega^2(R\sin\theta) \Rightarrow \quad \omega = \sqrt{\frac{g}{R\cos\theta}}$$

(a) The smallest value θ can have is 0 rad, in which case $\cos\theta = 1$ and

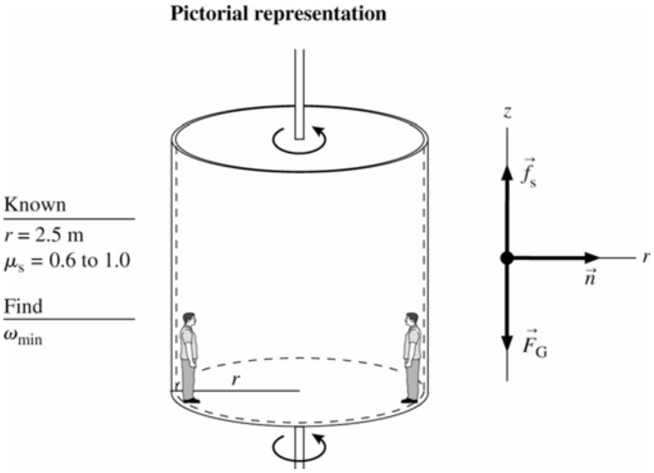
$$\omega_{\text{c}} = \sqrt{\frac{g}{R}} = \sqrt{\frac{(9.8\text{ m/s}^2)}{0.050\text{ m}}} = 14\text{ rad/s}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 130\text{ rpm}$$

(b) When we put in $\theta = 30°$ we get

$$\omega = \sqrt{\frac{g}{R\cos\theta}} = \sqrt{\frac{(9.8\text{ m/s}^2)}{(0.050\text{ m})\cos 30°}} = 15\text{ rad/s}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 140\text{ rpm}$$

Assess: Notice that in the other limiting case when $\theta \to 90°$ the angular speed is unbounded.

8.49. Model: Consider the passenger to be a particle and use the model of static friction.
Visualize:



Solve: The passengers stick to the wall if the static friction force is sufficient to support the gravitational force on them: $f_s = F_G$. The minimum angular velocity occurs when static friction reaches its maximum possible value $(f_s)_{\max} = \mu_s n.$ Although clothing has a range of coefficients of friction, it is the clothing with the smallest coefficient $(\mu_s = 0.60)$ that will slip first, so this is the case we need to examine. Assuming that the person is stuck to the wall, Newton's second law is

$$\sum F_r = n = m\omega^2 r \qquad \sum F_z = f_s - w = 0 \Rightarrow f_s = mg$$

The minimum frequency occurs when

$$f_s = (f_s)_{\max} = \mu_s n = \mu_s m r \omega_{\min}^2$$

Using this expression for f_s in the *z*-equation gives

$$f_s = \mu_s m r \omega_{\min}^2 = mg$$

$$\Rightarrow \omega_{\min} = \sqrt{\frac{g}{\mu_s r}} = \sqrt{\frac{9.80\text{ m/s}^2}{0.60(2.5\text{ m})}} = 2.56\text{ rad/s} = 2.56\text{ rad/s} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 24\text{ rpm}$$

Assess: Note the velocity does not depend on the mass of the individual. Therefore, the minimum mass sign is not necessary.

8.50. **Model:** Assume uniform circular motion.
Visualize: We expect the centripetal acceleration to be very large because ω is large. This will produce a significant force even though the mass difference of 10 mg is so small.

A preliminary calculation will convert the mass difference to kg: $10 \text{ mg} = 1.0 \times 10^{-5} \text{ kg}$. If the two samples are equally balanced then the shaft doesn't feel a net force in the horizontal plane. However, the mass difference of 10 mg is what causes the force.
We'll do another preliminary calculation to convert $\omega = 70{,}000 \text{ rpm}$ into rad/s.

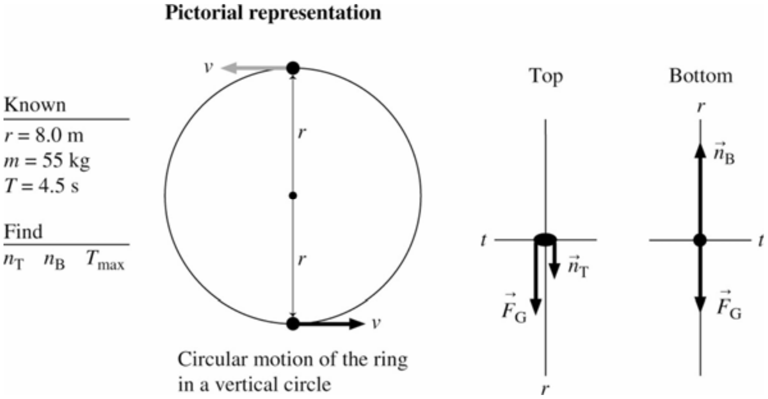
$$\omega = 70{,}000 \text{ rpm} = 70{,}000 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 7330 \text{ rad/s}$$

Solve: The centripetal acceleration is given by Equation 6.9 and the net force by Newton's second law.

$$F_{\text{net}} = (\Delta m)(a) = (\Delta m)(\omega^2 r) = (1.0 \times 10^{-5} \text{ kg})(7330 \text{ rad/s})^2(0.12 \text{ m}) = 64 \text{ N}$$

Assess: As we expected, the centripetal acceleration is large. The force is not huge (because of the small mass difference) but still enough to worry about. The net force scales with this mass difference, so if the mistake were bigger it could be enough to shear off the shaft

8.51. **Model:** Model a passenger as a particle rotating in a vertical circle.
Visualize:



Solve: **(a)** Newton's second law at the top is

$$\sum F_r = n_T + F_G = ma_r = \frac{mv^2}{r} \Rightarrow n_T + mg = \frac{mv^2}{r}$$

The speed is

$$v = \frac{2\pi r}{T} = \frac{2\pi(8.0 \text{ m})}{4.5 \text{ s}} = 11.17 \text{ m/s}$$

$$\Rightarrow n_T = m\left(\frac{v^2}{r} - g\right) = (55 \text{ kg})\left[\frac{(11.17 \text{ m/s})^2}{8.0 \text{ m}} - 9.8 \text{ m/s}^2\right] = 319 \text{ N}$$

That is, the ring pushes on the passenger with a force of 3.2×10^2 N at the top of the ride. Newton's second law at the bottom:

$$\sum F_r = n_B - F_G = ma_r = \frac{mv^2}{r} \Rightarrow n_B = \frac{mv^2}{r} + mg = m\left(\frac{v^2}{r} + g\right)$$

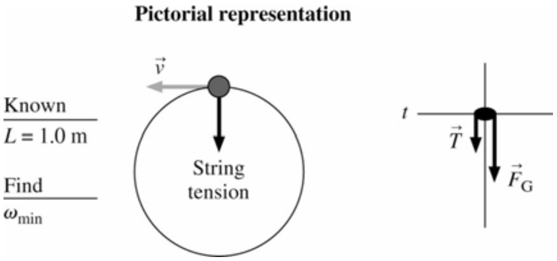
$$= (55 \text{ kg})\left[\frac{(11.17 \text{ m/s})^2}{8.0 \text{ m}} + 9.8 \text{ m/s}^2\right] = 1397 \text{ N}$$

Thus the force with which the ring pushes on the rider when she is at the bottom of the ring is 1.4 kN.

(b) To just stay on at the top, $n_\text{T} = 0\text{ N}$ in the *r*-equation at the top in part **(a)**. Thus,

$$mg = \frac{mv^2}{r} = mr\omega^2 = mr\left(\frac{2\pi}{T_\text{max}}\right)^2 \Rightarrow T_\text{max} = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{8.0\text{ m}}{9.8\text{ m/s}^2}} = 5.7\text{ s}$$

8.52. Model: Model the ball as a particle in motion in a vertical circle.
Visualize:



Solve: (a) *If* the ball moves in a complete circle, then there is a tension force $\vec{T}$ when the ball is at the top of the circle. The tension force adds to the gravitational force to cause the centripetal acceleration. The forces are along the *r*-axis, and the center of the circle is below the ball. Newton's second law at the top is

$$(F_\text{net})_r = T + F_\text{G} = T + mg = \frac{mv^2}{L}$$

$$\Rightarrow v_\text{top} = \sqrt{Lg + \frac{LT}{m}}$$

The tension *T* can't become negative, so $T = 0\text{ N}$ gives the minimum speed v_min at which the ball moves in a circle. If the speed is less than v_min, then the string will go slack and the ball will fall out of the circle before it reaches the top. Thus,

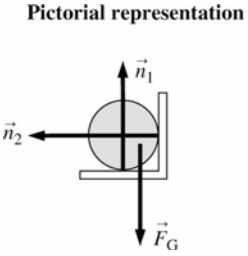
$$v_\text{min} = \sqrt{Lg} \Rightarrow \omega_\text{min} = \frac{v_\text{min}}{L} = \frac{\sqrt{Lg}}{L} = \sqrt{\frac{g}{L}}$$

(b) Insert *L* = 1.0 m.

$$\omega_\text{min} = \sqrt{\frac{g}{r}} = \sqrt{\frac{(9.8\text{ m/s}^2)}{(1.0\text{ m})}} = 3.13\text{ rad/s} = 30\text{ rpm}$$

Assess: Notice that the mass doesn't appear in the answer, so ω_min is independent of the mass.

8.53. Model: Model the ball as a particle in uniform circular motion. Rolling friction is ignored.
Visualize:



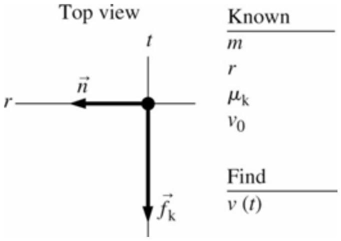
Solve: The track exerts both an upward normal force and an inward normal force. From Newton's second law,

$$n_1 = mg = (0.030\text{ kg})(9.8\text{ m/s}^2) = 0.294\text{ N, up}$$

$$n_2 = mr\omega^2 = (0.030\text{ kg})(0.20\text{ m})\left[\frac{60\text{ rev}}{\text{min}}\times\frac{2\pi\text{ rad}}{1\text{ rev}}\times\frac{1\text{ min}}{60\text{ s}}\right]^2 = 0.2369\text{ N, in}$$

$$F_{\text{net}} = \sqrt{n_1^2+n_2^2} = \sqrt{(0.294\text{ N})^2+(0.2369\text{ N})^2} = 0.38\text{ N}$$

8.54. Model: Model the block as a particle in circular motion.
Visualize: Apply Newton's second law in the *r*- and *t*-directions. We seek $v(t)$ where v is the tangential speed.



Solve:

$$\sum F_r = n = ma_r = \frac{mv^2}{r} \qquad \sum F_t = -f_k = ma_t$$

Use $n = mv^2/r$ in $f_k = \mu_k n.$

$$-f_k = -\mu_k n = -\mu_k\frac{mv^2}{r} = ma_t$$

Cancel m and use $a_t = \dfrac{dv}{dt}.$

$$-\frac{\mu_k}{r}v^2 = \frac{dv}{dt}$$

Separate variables and integrate.

$$-\frac{\mu_k}{r}\int_0^t dt = \int_{v_0}^{v}\frac{dv}{v^2}$$

$$-\frac{\mu_k}{r}[t]_0^t = \left[\frac{-1}{v}\right]_{v_0}^{v}$$

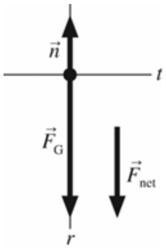
$$-\frac{\mu_k}{r}t = \frac{1}{v_0}-\frac{1}{v}$$

Solve for v which is a function of $t.$

$$\frac{1}{v} = \frac{1}{v_0}+\frac{\mu_k}{r}t \Rightarrow v = \left(\frac{1}{v_0}+\frac{\mu_k}{r}t\right)^{-1} = \frac{rv_0}{r+v_0\mu_k t}$$

Assess: The dependencies seem to go in the right direction: as t increases $v(t)$ decreases; as μ_k increases $v(t)$ decreases. The m canceled out, so the result does not depend on the mass.

8.55. Model: Model yourself as a particle in circular motion atop a leg of length L.
Visualize: Set up the coordinate system so that the r-direction is down, toward the center of the circle.



Solve:
(a) Apply Newton's second law in the downward vertical direction, which is the $+r$-direction.

$$\sum F_r = mg - n = \frac{mv^2}{L}$$

Notice that $n < mg$. Your body tries to "lift off" as it pivots over your foot, decreasing the normal force exerted on you by the ground. The normal force becomes smaller as you walk faster, but n cannot be less than zero. Thus the maximum possible walking speed v_{max} occurs when $n = 0$. Setting $n = 0$,

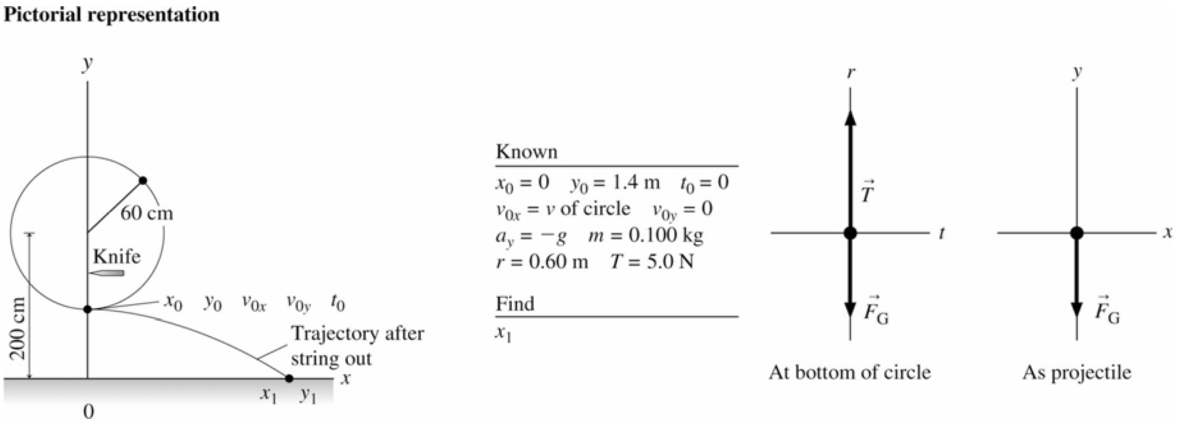
$$mg = \frac{mv_{max}^2}{L} \Rightarrow v_{max} = \sqrt{gL}$$

(b) Insert $L = 0.70\text{ m}$.

$$v_{max} = \sqrt{gL} = \sqrt{(9.8\text{ m/s}^2)(0.70\text{ m})} = 2.6\text{ m/s} = 5.9\text{ mph}$$

Assess: The answer of 5.9 mph is faster than the "normal" walking speed of 3 mph, but we expect v_{max} to be greater than the normal speed. This seems reasonable. The units check out.
The maximum walking speed depends on L, so taller people can walk faster than shorter people.

8.56. Model: Model the ball as a particle swinging in a vertical circle, then as a projectile.
Visualize:



Known
$x_0 = 0$ $y_0 = 1.4\text{ m}$ $t_0 = 0$
$v_{0x} = v$ of circle $v_{0y} = 0$
$a_y = -g$ $m = 0.100\text{ kg}$
$r = 0.60\text{ m}$ $T = 5.0\text{ N}$

Find
x_1

Solve: Initially, the ball is moving in a circle. Once the string is cut, it becomes a projectile. The final circular-motion velocity is the initial velocity for the projectile. The free-body diagram for circular motion is shown at the bottom of the circle. Since $T > F_G$, there is a net force toward the center of the circle that causes the centripetal acceleration. The r-equation of Newton's second law is

$$(F_{\text{net}})_r = T - F_G = T - mg = \frac{mv^2}{r}$$

$$\Rightarrow v_{\text{bottom}} = \sqrt{\frac{r}{m}(T - mg)} = \sqrt{\frac{0.60\text{ m}}{0.100\text{ kg}}[5.0\text{ N} - (0.10\text{ kg})(9.8\text{ m/s}^2)]} = 4.91\text{ m/s}$$

As a projectile the ball starts at $y_0 = 1.4$ m with $\vec{v}_0 = 4.91\hat{i}$ m/s. The equation for the y-motion is

$$y_1 = 0\text{ m} = y_0 + v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2 = y_0 - \tfrac{1}{2}gt_1^2$$

This is easily solved to find that the ball hits the ground at time

$$t_1 = \sqrt{\frac{2y_0}{g}} = 0.535\text{ s}$$

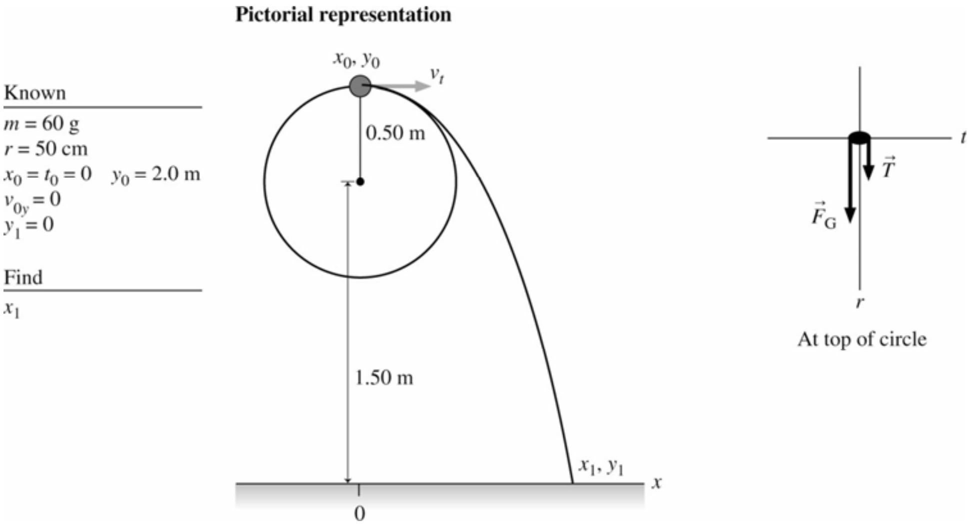
During this time interval it travels a horizontal distance

$$x_1 = x_0 + v_{0x}t_1 = (4.91\text{ m/s})(0.535\text{ s}) = 2.63\text{ m}$$

So the ball hits the floor 2.6 m to the right of the point where the string was cut.

8.57. Model: Use the particle model for a ball in motion in a vertical circle and then as a projectile.

Visualize:



Solve: For the circular motion, Newton's second law along the r-direction is

$$\sum F_r = T + F_G = \frac{mv_t^2}{r}$$

Since the string goes slack as the particle makes it over the top, $T = 0$ N. That is,

$$F_G = mg = \frac{mv_t^2}{r} \Rightarrow v_t = \sqrt{gr} = \sqrt{(9.8\text{ m/s}^2)(0.5\text{ m})} = 2.21\text{ m/s}$$

The ball begins projectile motion as the string is released. The time it takes for the ball to hit the floor can be found as follows:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \Rightarrow 0\text{ m} = 2.0\text{ m} + 0\text{ m} + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t_1 - 0\text{ s})^2 \Rightarrow t_1 = 0.639\text{ s}$$

The place where the ball hits the ground is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) = 0\text{ m} + (+2.21\text{ m/s})(0.639\text{ s} - 0\text{ s}) = +1.41\text{ m}$$

The ball hits the ground 1.4 m to the right of the point beneath the center of the circle.

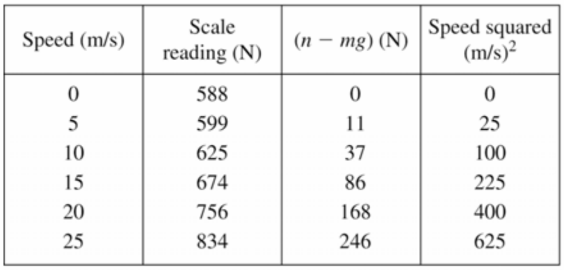
8.58. Model: Model yourself as a particle in circular motion.

Visualize: Apply Newton's second law for circular motion in the vertical direction. The upward normal force is the scale reading. We are given $mg = 588\text{ N}$. This means $m = 60\text{ kg}$. We seek r.

Solve:

$$\sum F = n - mg = \frac{mv^2}{r}$$

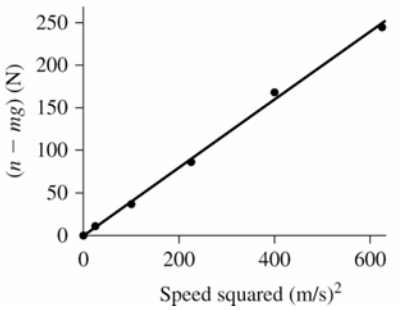
We can easily get the spreadsheet to subtract $mg = 588\text{ N}$ from each of the scale readings on the left side of the equation. Then if we graph $n - mg$ vs. v^2 we should get a straight line whose slope is m/r.



Speed (m/s)	Scale reading (N)	$(n - mg)$ (N)	Speed squared $(\text{m/s})^2$
0	588	0	0
5	599	11	25
10	625	37	100
15	674	86	225
20	756	168	400
25	834	246	625

$n - mg$ vs. speed squared

$y = 0.3999x - 0.3205$, $R^2 = 0.9978$



We see from the spreadsheet that the fit is very good and that the slope is $m/r = 0.3999\text{ kg/m}$.

With $m = 60\text{ kg}$,

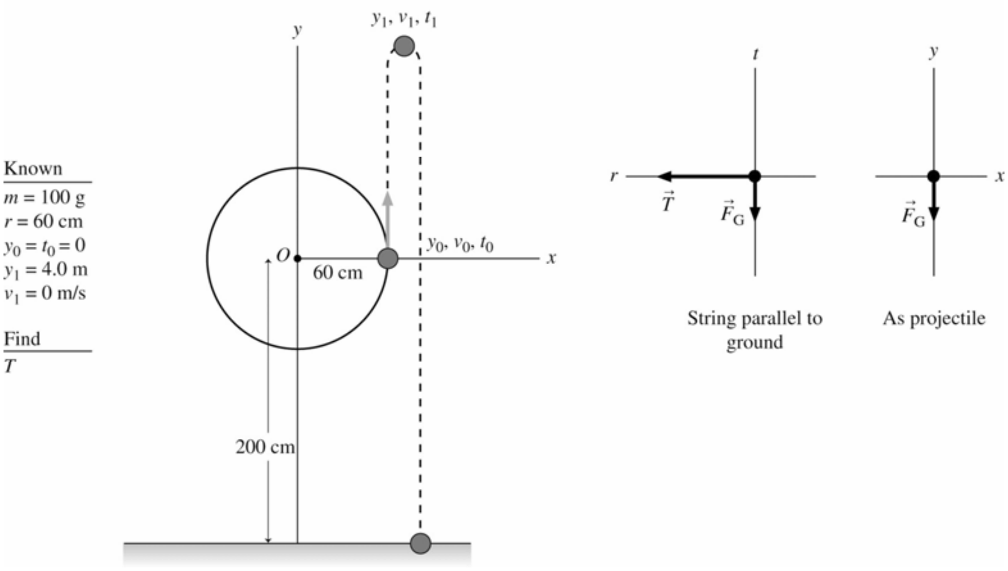
$$\frac{m}{r} = 0.3999\text{ kg/m} \Rightarrow r = \frac{60\text{ kg}}{0.3999\text{ kg/m}} = 150\text{ m}$$

Assess: This radius of curvature does not seem extreme.

8.59. Model: Model the ball as a particle undergoing circular motion in a vertical circle.

Visualize:

Pictorial representation



Solve: Initially, the ball is moving in circular motion. Once the string breaks, it becomes a projectile. The final circular-motion velocity is the initial velocity for the projectile, which we can find by using the kinematic equation

$$v_1^2 = v_0^2 + 2a_y(y_1 - y_0) \Rightarrow 0\ \text{m}^2/\text{s}^2 = (v_0)^2 + 2(-9.8\ \text{m/s}^2)(4.0\ \text{m} - 0\ \text{m}) \Rightarrow v_0 = 8.85\ \text{m/s}$$

This is the speed of the ball as the string broke. The tension in the string at that instant can be found by using the *r*-component of the net force on the ball:

$$\sum F_r = T = m\left(\frac{v_{0y}^2}{r}\right) \Rightarrow T = (0.100\ \text{kg})\frac{(8.85\ \text{m/s})^2}{0.60\ \text{m}} = 13\ \text{N}$$

8.60. Model: The particle is in nonuniform circular motion. The particle's speed is increasing (since that is what particle accelerators do).

Visualize: The smallest value of the sine or cosine is 0 and the largest value is 1, regardless of what is in the argument.

Solve: (a) When the sine is 0 the cosine is 1, so we see the radius of the circle is $c = 5.0$ m.

(b) We take the derivative of the position to get the velocity and plug in $t = 3.0$ s.

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d(c\cos(kt^2)\hat{i} + c\sin(kt^2)\hat{j})}{dt} = -c(2kt)\sin(kt^2)\hat{i} + c(2kt)\cos(kt^2)\hat{j}$$

The speed is the magnitude of this vector so square both components, add them, and take the square root.

$$v = \sqrt{(-c(2kt)\sin(kt^2))^2 + (c(2kt)\cos(kt^2))^2} = 2ckt\sqrt{\sin^2(kt^2) + \cos^2(kt^2)} = 2ckt\sqrt{1}$$

Inert the values given.

$$v = 2\,ckt = 2(5.0\ \text{m})(8.0\times10^4\ \text{rad/s}^2)(3.0\ \text{s}) = 2.4\times10^6\ \text{m/s}$$

(c) To find the force we first take the second derivative to get the acceleration.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(-c(2kt)\sin(kt^2)\hat{i} + c(2kt)\cos(kt^2)\hat{j})}{dt} = -c(2kt)^2\cos(kt^2)\hat{i} - c(2kt)^2\sin(kt^2)\hat{j}$$
$$= -c(2kt)^2[\cos(kt^2)\hat{i} + \sin(kt^2)\hat{j}]$$

Some intermediate calculations:

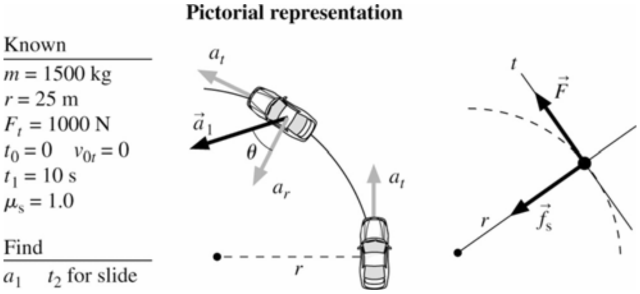
$$-c(2kt)^2 = -(5.0\ \text{m})(2(8.0\times10^4\ \text{rad/s}^2)(3.0\ \text{s}))^2 = -1.15\times10^{12}\ \text{m/s}^2$$
$$kt^2 = (8.0\times10^4\ \text{rad/s}^2)(3.0\ \text{s})^2 = 720000\ \text{rad}$$

$$F = ma = (1.7\times10^{-27}\ \text{kg})(-1.15\times10^{12}\ \text{m/s}^2)[\cos(720000\ \text{rad})\hat{i} + \sin(720000\ \text{rad})\hat{j}]$$
$$= (1.8\times10^{-15}\hat{i} + 7.1\times10^{-16}\hat{j})\ \text{N}$$

Assess: This is a tiny force, but the proton is so tiny that it still produces a large acceleration.

8.61. Model: Model the car as a particle on a circular track.

Visualize:



Solve: **(a)** Newton's second law along the *t*-axis is

$$\sum F_t = F_t = ma_t \Rightarrow 1000\ \text{N} = (1500\ \text{kg})a_t \Rightarrow a_t = 2/3\ \text{m/s}^2$$

With this tangential acceleration, the car's tangential velocity after 10 s will be

$$v_{1t} = v_{0t} + a_t(t_1 - t_0) = 0\ \text{m/s} + (2/3\ \text{m/s}^2)(10\ \text{s} - 0\ \text{s}) = 20/3\ \text{m/s}$$

The radial acceleration at this instant is

$$a_r = \frac{v_{1t}^2}{r} = \frac{(20/3\ \text{m/s})^2}{25\ \text{m}} = \frac{16}{9}\ \text{m/s}^2$$

The car's acceleration at 10 s has magnitude

$$a_1 = \sqrt{a_t^2 + a_r^2} = \sqrt{(2/3\ \text{m/s}^2)^2 + (16/9\ \text{m/s}^2)^2} = 1.90\ \text{m/s}^2 \qquad \theta = \tan^{-1}\frac{a_t}{a_r} = \tan^{-1}\left(\frac{2/3}{16/9}\right) = 21^\circ$$

where the angle is measured from the *r*-axis.

(b) The car will begin to slide out of the circle when the static friction reaches its maximum possible value $(f_\text{s})_\text{max} = \mu_\text{s} n$. That is,

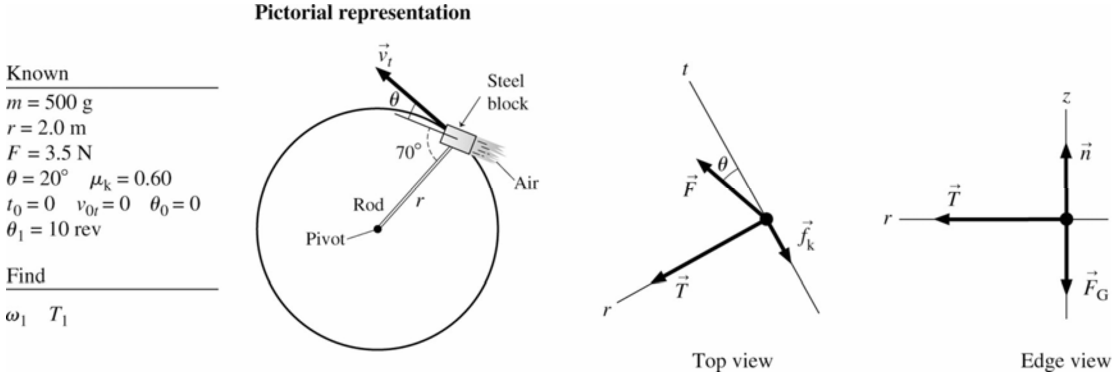
$$\sum F_r = (f_\text{s})_\text{max} = \mu_\text{s} n = \mu_\text{s} mg = \frac{mv_{2t}^2}{r} \Rightarrow v_{2t} = \sqrt{rg} = \sqrt{(25\ \text{m})(9.8\ \text{m/s}^2)} = 15.7\ \text{m/s}$$

In the above equation, $n = mg$ follows from Newton's second law along the *z*-axis. The time when the car begins to slide can now be obtained as follows:

$$v_{2t} = v_{0t} + a_t(t_2 - t_0) \Rightarrow 15.7\ \text{m/s} = 0\ \text{m/s} + (2/3\ \text{m/s}^2)(t_2 - 0) \Rightarrow t_2 = 24\ \text{s}$$

8.62. **Model:** Model the steel block as a particle and use the model of kinetic friction.

Visualize:



Solve: **(a)** The components of thrust $(\vec{F})$ along the *r*-, *t*-, and *z*-directions are

$$F_r = F\sin 20^\circ = (3.5\ \text{N})\sin 20^\circ = 1.20\ \text{N} \qquad F_t = F\cos 20^\circ = (3.5\ \text{N})\cos 20^\circ = 3.29\ \text{N} \qquad F_z = 0\ \text{N}$$

Newton's second law is

$$(F_\text{net})_r = T + F_r = mr\omega^2 \quad (F_\text{net})_t = F_t - f_\text{k} = ma_t$$

$$(F_\text{net})_z = n - mg = 0\ \text{N}$$

The *z*-component equation means $n = mg$. The force of friction is

$$f_\text{k} = \mu_\text{k} n = \mu_\text{k} mg = (0.60)(0.500\ \text{kg})(9.8\ \text{m/s}^2) = 2.94\ \text{N}$$

Substituting into the *t*-component of Newton's second law

$$(3.29\ \text{N}) - (2.94\ \text{N}) = (0.500\ \text{kg})a_t \Rightarrow a_t = 0.70\ \text{m/s}^2$$

Having found a_t, we can now find the tangential velocity after $10 \text{ revolutions} = 20\pi$ rad as follows:

$$\theta_1 = \frac{1}{2}\left(\frac{a_t}{r}\right)t_1^2 \Rightarrow t_1 = \sqrt{\frac{2r\theta_1}{a_t}} = 18.95 \text{ s}$$

$$\omega_1 = \omega_0 + \left(\frac{a_t}{r}\right)t_1 = 6.63 \text{ rad/s}$$

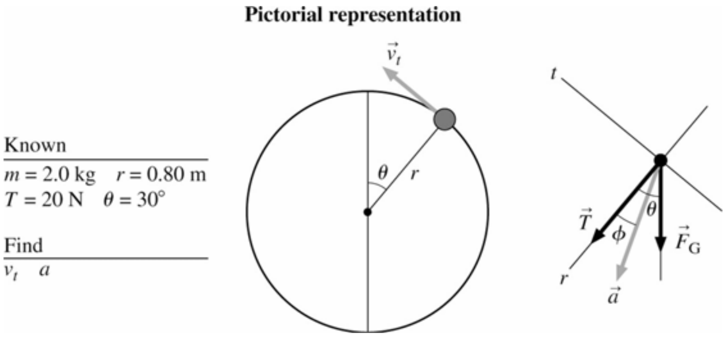
The block's angular velocity after 10 rev is 6.6 rad/s.

(b) Substituting ω_1 into the *r*-component of Newton's second law yields:

$$T_1 + F_r = mr\omega_1^2 \Rightarrow T_1 + (1.20 \text{ N}) = (0.500 \text{ kg})(2.0 \text{ m})(6.63 \text{ rad/s})^2 \Rightarrow T_1 = 44 \text{ N}$$

8.63. Model: Assume the particle model for a ball in vertical circular motion.

Visualize:



Solve: (a) Newton's second law in the *r*- and *t*-directions is

$$(F_{\text{net}})_r = T + mg\cos\theta = ma_r = \frac{mv_t^2}{r} \quad (F_{\text{net}})_t = -mg\sin\theta = ma_t$$

Substituting into the *r*-component,

$$(20 \text{ N}) + (2.0 \text{ kg})(9.8 \text{ m/s}^2)\cos 30° = (2.0 \text{ kg})\frac{v_t^2}{(0.80 \text{ m})} \Rightarrow v_t = 3.85 \text{ m/s}$$

The tangential velocity is 3.8 m/s.

(b) Substituting into the *t*-component,

$$-(9.8 \text{ m/s}^2)\sin 30° = a_t \Rightarrow a_t = -4.9 \text{ m/s}^2$$

The radial acceleration is

$$a_r = \frac{v_t^2}{r} = \frac{(3.85 \text{ m/s})^2}{0.80 \text{ m}} = 18.5 \text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(18.5 \text{ m/s}^2)^2 + (-4.9 \text{ m/s}^2)^2} = 19.1 \text{ m/s}^2 \approx 19 \text{ m/s}^2$$

The angle of the acceleration vector from the *r*-axis is

$$\phi = \tan^{-1}\frac{|a_t|}{a_r} = \tan^{-1}\frac{4.9}{18.5} = 14.8° \approx 15°$$

The angle is below the *r*-axis.

8.64. Solve: (a) You are spinning a lead fishing weight in a horizontal 1.0 m diameter circle on the ice of a pond when the string breaks. You know that the test weight (breaking force) of the line is 60 N and that the lead weight has a mass of 0.30 kg. What was the weight's angular velocity in rad/s and in rpm?

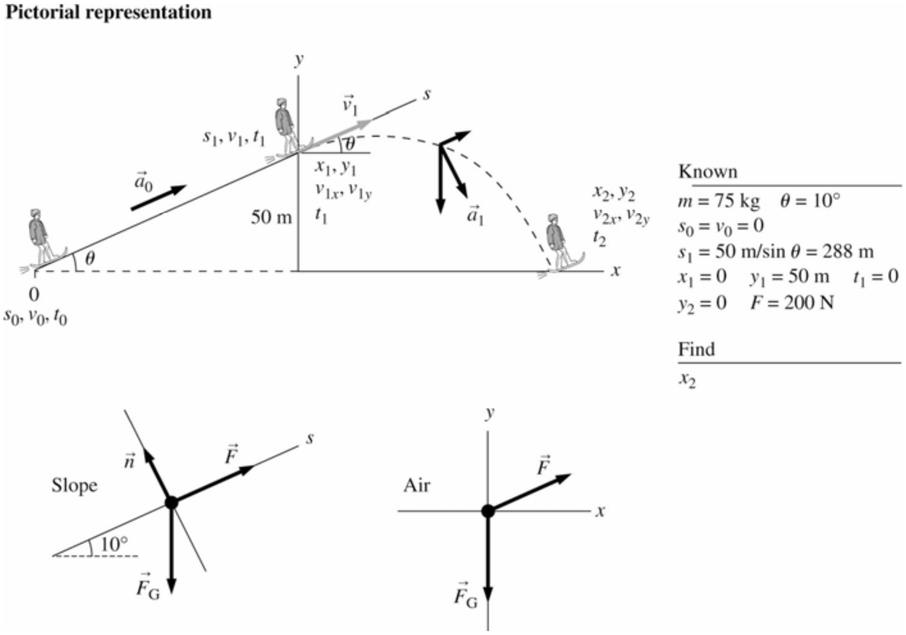
(b)

$$\omega^2 = \frac{60\text{ N}}{(0.3\text{ kg})(0.5\text{ m})} \Rightarrow \omega = 20\text{ rad/s } \times \frac{\text{rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{\text{min}} = 191\text{ rpm}$$

8.65. Solve: (a) At what speed does a 1500 kg car going over a hill with a radius of 200 m have a weight of 11,760 N?
(b) The weight is the normal force.

$$2940\text{ N} = \frac{1500\text{ kg } v^2}{200\text{ m}} \Rightarrow v = 19.8\text{ m/s}$$

8.66. Model: Treat Sam as a particle.
Visualize: This is a two-part problem. Use an *s*-axis parallel to the slope for the first part, regular *xy*-coordinates for the second. Sam's final velocity at the top of the slope is his initial velocity as he becomes airborne.


Pictorial representation

Solve: Sam's acceleration up the slope is given by Newton's second law:

$$(F_{\text{net}})_s = F - mg\sin 10° = ma_0$$

$$a_0 = \frac{F}{m} - g\sin 10° = \frac{200\text{ N}}{75\text{ kg}} - (9.8\text{ m/s}^2)\sin 10° = 0.965\text{ m/s}^2$$

The length of the slope is $s_1 = (50\text{ m})/\sin 10° = 288\text{ m}.$ His velocity at the top of the slope is

$$v_1^2 = v_0^2 + 2a_0(s_1 - s_0) = 2a_0 s_1 \Rightarrow v_1 = \sqrt{2(0.965\text{ m/s}^2)(288\text{ m})} = 23.6\text{ m/s}$$

This is Sam's initial speed into the air, giving him velocity components $v_{1x} = v_1\cos 10° = 23.2\text{ m/s}$ and $v_{1y} = v_1\sin 10° = 410\text{ m/s}.$ This is not projectile motion because Sam experiences both the force of gravity *and* the thrust of his skis. Newton's second law for Sam's acceleration is

$$a_{1x} = \frac{(F_{\text{net}})_x}{m} = \frac{(200\text{ N})\cos 10°}{75\text{ kg}} = 2.63\text{ m/s}^2$$

$$a_{1y} = \frac{(F_{\text{net}})_y}{m} = \frac{(200\text{ N})\sin 10° - (75\text{ kg})(9.80\text{ m/s}^2)}{75\text{ kg}} = -9.34\text{ m/s}^2$$

The y-equation of motion allows us to find out how long it takes Sam to reach the ground:

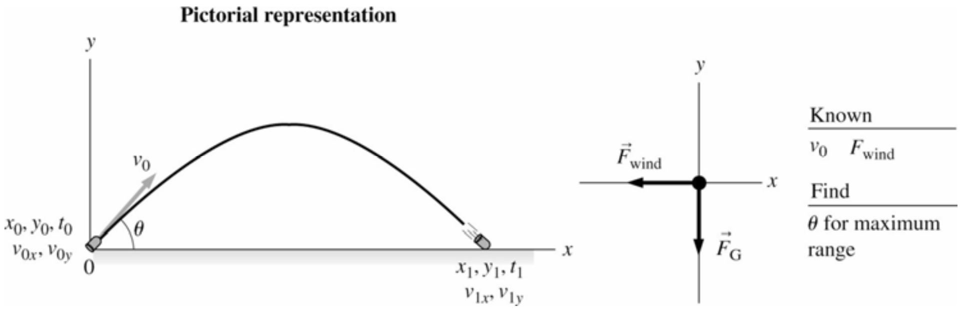
$$y_2 = 0\text{ m} = y_1 + v_{1y}t_2 + \tfrac{1}{2}a_{1y}t_2^2 = 50\text{ m} + (4.10\text{ m/s})t_2 - (4.67\text{ m/s}^2)t_2^2$$

This quadratic equation has roots $t_2 = -2.86$ s (unphysical) and $t_2 = 3.74$ s. The x-equation of motion—this time with an acceleration—is

$$x_2 = x_1 + v_{1x}t_2 + \tfrac{1}{2}a_{1x}t_2^2 = 0\text{ m} + (23.2\text{ m/s})t_2 - \tfrac{1}{2}(2.63\text{ m/s}^2)t_2^2 = 105\text{ m}$$

Sam lands 105 m from the base of the cliff.

8.67. Model: Assume the particle model and apply the constant-acceleration kinematic equations.
Visualize:



Solve: (a) Newton's second law for the projectile is

$$(\vec{F}_{\text{net}})_x = -F_{\text{wind}} = ma_x \Rightarrow a_x = \frac{-F}{m}$$

where F_{wind} is shortened to F. For the y-motion:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \Rightarrow 0\text{ m} = 0\text{ m} + (v_0\sin\theta)t_1 - \tfrac{1}{2}gt_1^2 \Rightarrow t_1 = 0\text{ s and } t_1 = \frac{2v_0\sin\theta}{g}$$

Using the above expression for t_1 and defining the range as R we get from the x motion:

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2$$

$$\Rightarrow x_1 - x_0 = R = v_{0x}t_1 + \tfrac{1}{2}\left(-\frac{F}{m}\right)t_1^2 = (v_0\cos\theta)\left(\frac{2v_0\sin\theta}{g}\right) - \frac{F}{2m}\left(\frac{2v_0\sin\theta}{g}\right)^2$$

$$= \frac{2v_0^2}{g}\cos\theta\sin\theta - \frac{2v_0^2F}{mg^2}\sin^2\theta$$

We will now maximize R as a function of θ by setting the derivative equal to 0:

$$\frac{dR}{d\theta} = \frac{2v_0^2}{g}(\cos^2\theta - \sin^2\theta) - \frac{2Fv_0^2}{mg^2}2\sin\theta\cos\theta = 0$$

$$\Rightarrow \cos^2\theta - \sin^2\theta = \cos 2\theta = \left(\frac{2Fv_0^2}{mg^2}\right)\left(\frac{g}{2v_0^2}\right)\sin 2\theta \Rightarrow \tan 2\theta = \frac{mg}{F}$$

Thus the angle for maximum range is $\theta = \tfrac{1}{2}\tan^{-1}(mg/F)$.

(b) We have

$$\frac{mg}{F} = \frac{(0.50\text{ kg})(9.8\text{ m/s}^2)}{0.60\text{ N}} = 8.167 \Rightarrow \theta = \tfrac{1}{2}\tan^{-1}(8.167) = 41.51°$$

The maximum range without air resistance is

$$R' = \frac{2v_0^2 \sin 45° \cos 45°}{g} = \frac{v_0^2}{g}$$

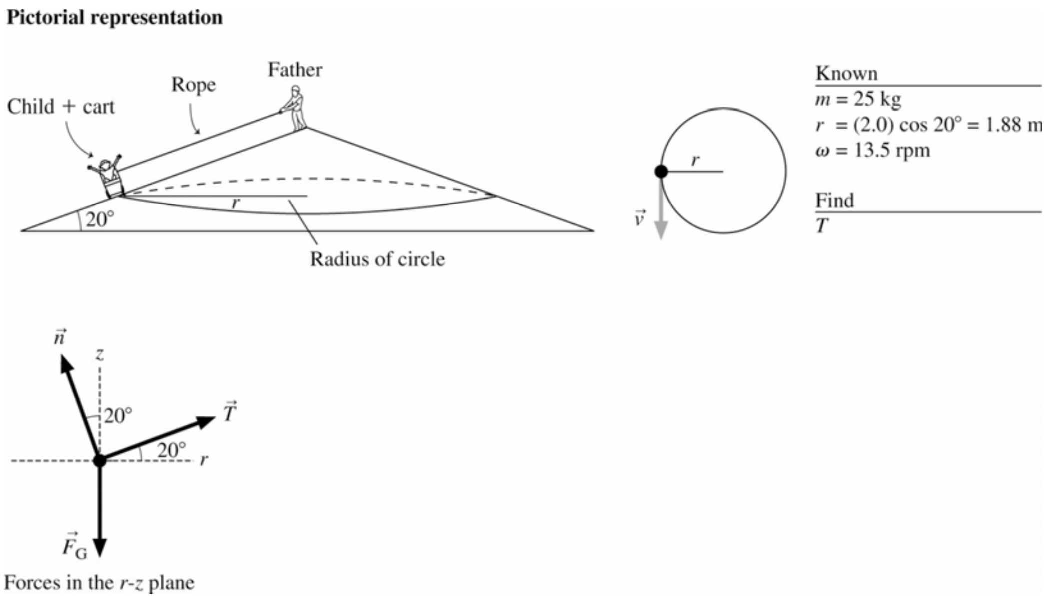
Therefore, we can write the equation for the range R as

$$R = 2R' \sin 41.51° \cos 41.51° - \frac{2F}{mg} R' \sin^2 41.51° = R'(0.9926 - 0.1076) = 0.885R'$$

$$\Rightarrow \frac{R}{R'} = 0.8850 \Rightarrow \frac{R' - R}{R'} = 1 - 0.8850 = 0.115$$

Thus R is reduced from R' by 11.5%.
Assess: The condition for maximum range $(\tan 2\theta = mg/F)$ means $2\theta \to 90°$ as $F \to 0$. That is, $\theta = 45°$ when $F = 0$, as is to be expected.

8.68. Model: Use the particle model for the (cart + child) system which is in uniform circular motion.
Visualize:



Solve: Newton's second law along r and z directions can be written:

$$\sum F_r = T\cos 20° - n\sin 20° = ma_r \quad \sum F_z = T\sin 20° - n\cos 20° - mg = 0$$

The cart's centripetal acceleration is

$$a_r = r\omega^2 = (2.0\cos 20° \text{ m})\left(14 \ \frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}}\right)^2 = 4.04 \text{ m/s}^2$$

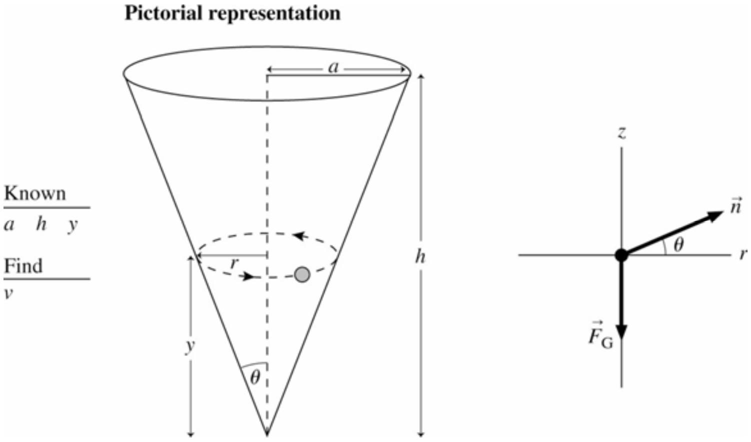
The above force equations can be rewritten as

$$0.94T - 0.342n = (25 \text{ kg})(4.04 \text{ m/s}^2) = 101 \text{ N}$$

$$0.342T + 0.94n = (25 \text{ kg})(9.8 \text{ m/s}^2) = 245 \text{ N}$$

Solving these two equations yields $T = 179 \text{ N} \approx 180 \text{ N}$ for the tension in the rope.
Assess: In view of the (child + cart) weight of 245 N, a tension of 179 N is reasonable.

8.69. Model: Use the particle model for the ball, which is in uniform circular motion.
Visualize:



Solve: From Newton’s second law along *r* and *z* directions,

$$\sum F_r = n\cos\theta = \frac{mv^2}{r} \qquad \sum F_z = n\sin\theta - mg = 0 \Rightarrow n\sin\theta = mg$$

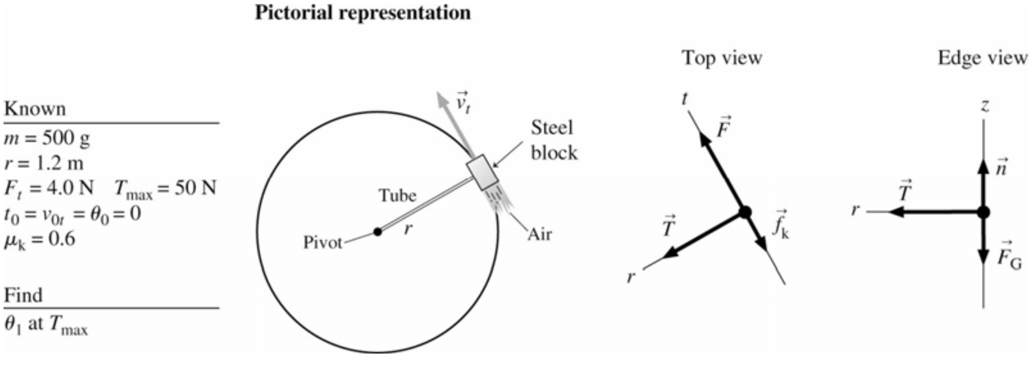
Dividing the two force equations gives

$$\tan\theta = \frac{gr}{v^2}$$

From the geometry of the cone, $\tan\theta = r/y$. Thus

$$\frac{r}{y} = \frac{gr}{v^2} \Rightarrow v = \sqrt{gy}$$

8.70. Model: Model the block as a particle and use the model of kinetic friction.
Visualize:



Solve: The only radial force is tension, so we can use Newton’s second law to find the angular velocity $\omega_{\max}$ at which the tube breaks:

$$\sum F_r = T = m\omega^2 r \Rightarrow \omega_{\max} = \sqrt{\frac{T_{\max}}{mr}} = \sqrt{\frac{50\text{ N}}{(0.50\text{ kg})(1.2\text{ m})}} = 9.12\text{ rad/s}$$

The compressed air and friction exert tangential forces, and the second law along the tangential direction is

$$\sum F_t = F_t - f_\text{k} = F_t - \mu_\text{k} n = F_t - \mu_\text{k} mg = ma_t$$

$$a_t = \frac{F_t}{m} - \mu_\text{k} g = \frac{4.0\text{ N}}{0.50\text{ kg}} - (0.60)(9.80\text{ m/s}^2) = 2.12\text{ m/s}^2$$

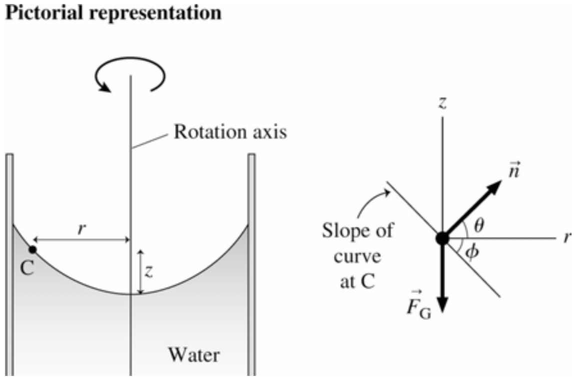
The time needed to accelerate to 9.12 rad/s is given by

$$\omega_1 = \omega_\text{max} = 0 + \left(\frac{a_t}{r}\right)t_1 \Rightarrow t_1 = \frac{r\omega_\text{max}}{a_t} = \frac{(1.2)(9.12\text{ rad/s})}{2.12\text{ m/s}^2} = 5.16\text{ s}$$

During this interval, the block turns through angle

$$\Delta\theta = \theta_1 - \theta_0 = \omega_0 t_1 + \tfrac{1}{2}\left(\frac{a_t}{r}\right)t_1^2 = 0 + \tfrac{1}{2}\left(\frac{2.12\text{ m/s}^2}{1.2\text{ m}}\right)(5.16\text{ s})^2 = 23.52\text{ rad} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = 3.7\text{ rev}$$

8.71. Model: Use the particle model for a small volume of water on the surface.
Visualize:


Pictorial representation

Solve: Consider a particle of water of mass *m* at point C on the surface. Newton's second law along the *r*- and *z*-directions is

$$(F_\text{net})_r = n\cos\theta = mr\omega^2 \Rightarrow \cos\theta = \frac{mr\omega^2}{n}$$

$$(F_\text{net})_z = n\sin\theta - mg = 0\text{ N} \Rightarrow \sin\theta = \frac{mg}{n}$$

Dividing both equations gives $\tan\theta = g/r\omega^2$. For a parabola $z = ar^2$. This means

$$\frac{dz}{dr} = 2ar = \text{slope of the curve at } C = \tan\phi = \tan(90° - \theta) = \frac{1}{\tan\theta} \Rightarrow \tan\theta = \frac{1}{2ar}$$

Equating the two equations for $\tan\theta$, we get

$$\frac{1}{2ar} = \frac{g}{r\omega^2} \Rightarrow a = \frac{\omega^2}{2g}$$

Thus the surface is described by the equation

$$z = \frac{\omega^2}{2g}r^2$$

which is the equation of a parabola.